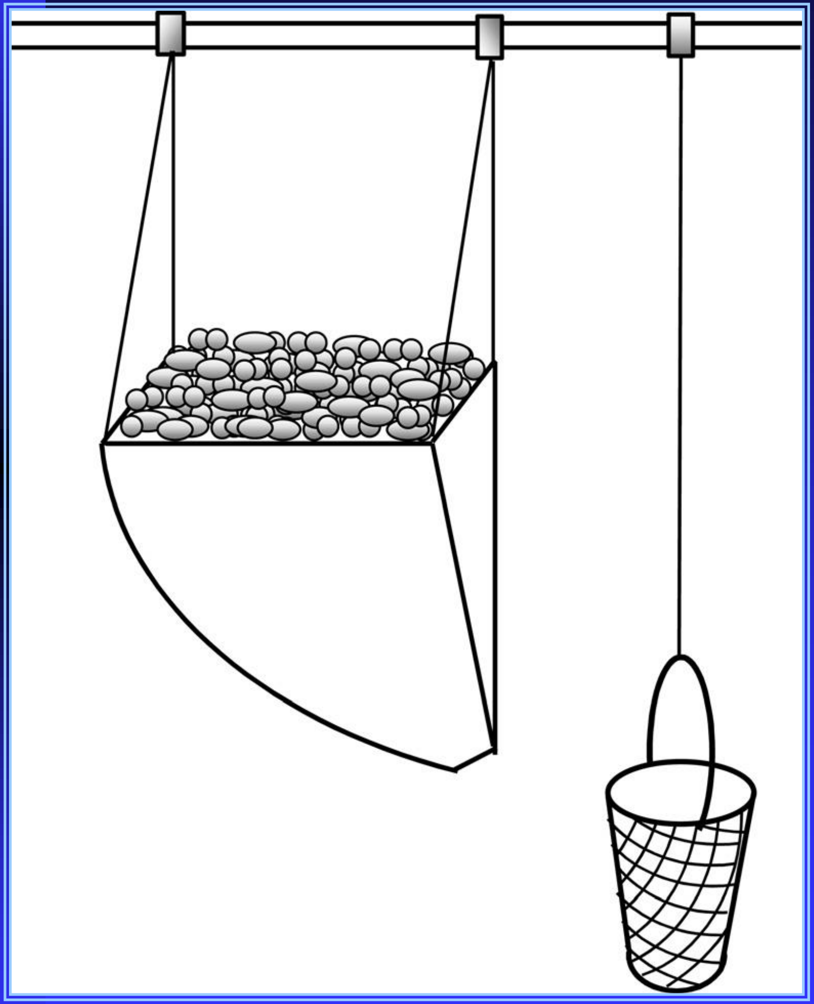
Elasticity

- Elastic deformation (springs, spring washers)
- Structure rigidity (undesirable flexure, change of geometry)
- Structure vibrations dumping (eigenfrequency)

R. Hook: „ut tensio, sic vis“ (1676)

Elasticity



- Leonardo da Vinci tensile test of wires
- 15^{th} century !


Tacoma Bridge Collapse

Elasticity

1) Elastic (Young's) modulus
2) Bonds between atoms
3) Atoms ordering in solids
4) Physical background of Youngs modulus

Inelasticity/internal dumping

1) Parameters introduction and measurement
2) Physical and practical meaning

Young’s modulus

- Elastic characteristics of materials
- Measurements of elastic characteristics
- Values of Young’s modulus

Generalized Hooke's law

Anisotropic material (Anisotropy = dependence of physical properties of materials on the direction in which this property is determined)

Elastic coefficients

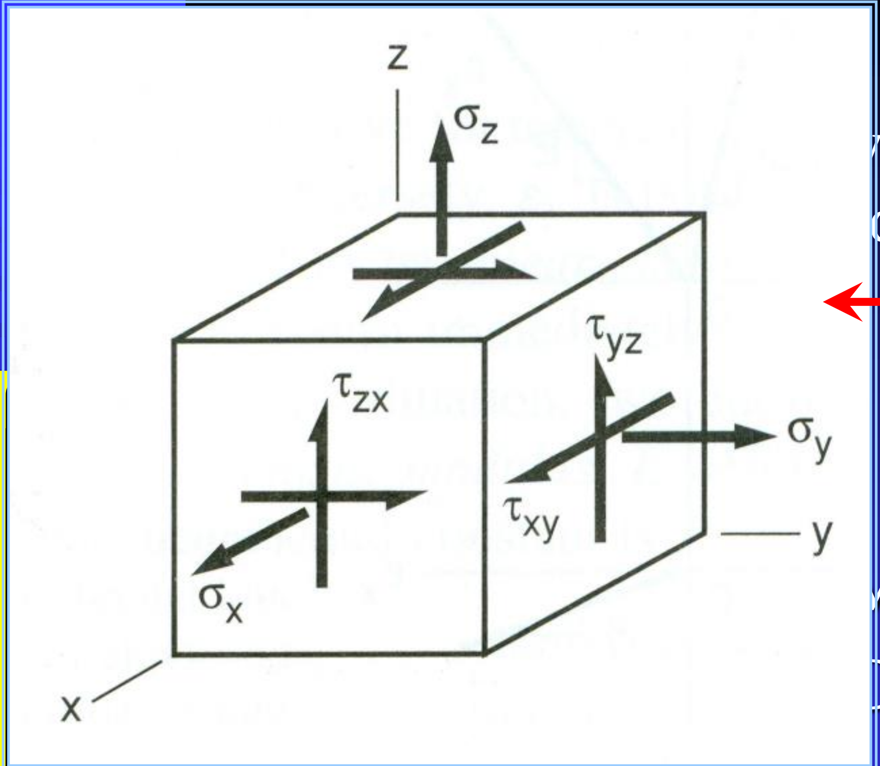
Strain tensor

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{Bmatrix} * \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix}$$

Stress tensor

The matrix contains 36 elements, however, thanks to indexes symmetry only 21 are independent

Generalized Hooke's law



(by introducing axis x,

dependent elastic

Orthotropic material

Orthotropy = right angle anisotropy

Monocrystal, texture

Composite – matrix + fiber

(axis „z“ is neglected)

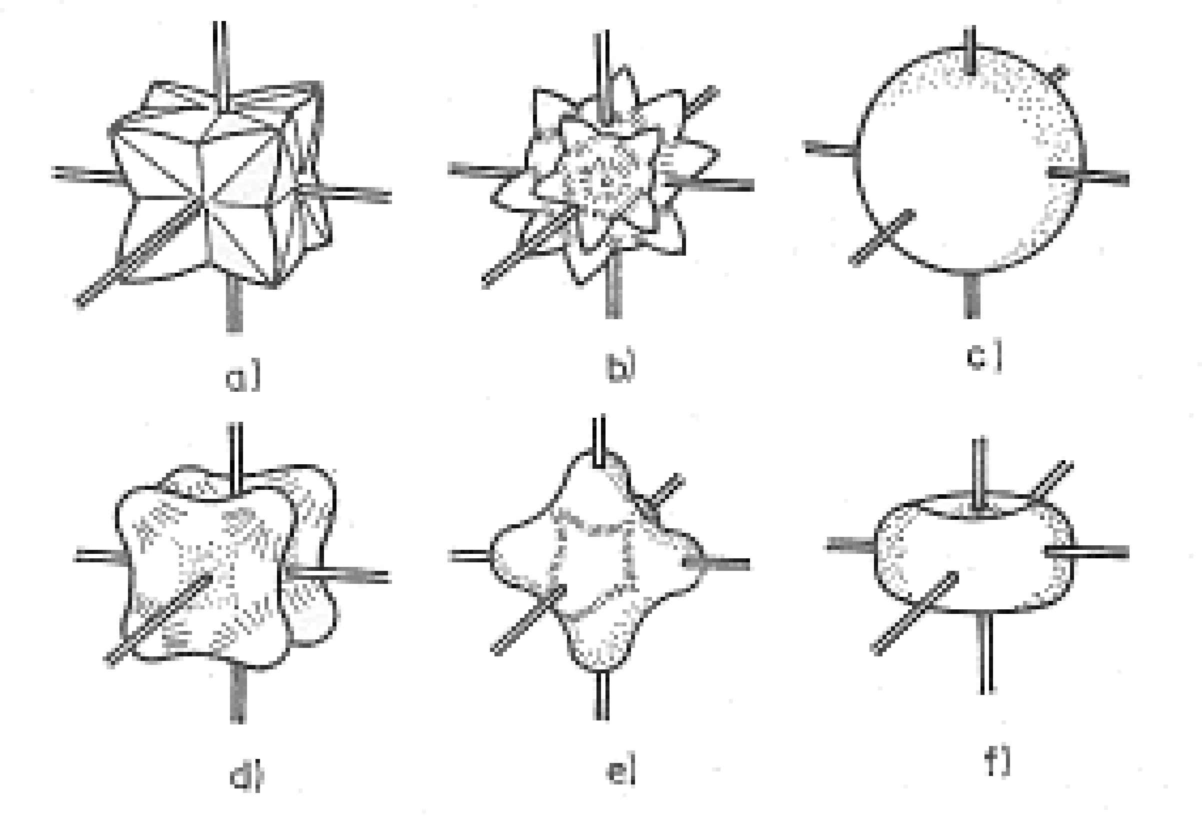
$$\left\{ \ldots \frac{}{E_x}, \frac{}{E_y}, \frac{}{E_z}, 0,0,0 \quad 0,0,0,\frac{1}{G_{yz}},0,0 \quad 0,0,0,0,\frac{1}{G_{zx}},0 \quad 0,0,0,0,0,\frac{1}{G_{xy}} \right\}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{E_x}, -\frac{\mu_{yx}}{E_y}, 0 \\ -\frac{\mu_{xy}}{E_x}, \frac{1}{E_y}, 0 \\ 0,0,\frac{1}{G_{xy}} \end{Bmatrix} * \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

Generalized Hooke's law

Orthotropic material

End points of Young's modulus vectors



monocrystals

a) Al (fract.stress)

b) Al (ductility)

c) Al (E)

d) Fe (E)

e) Fe (G)

f) Mg (E)

Isotropic material

Generalized Hooke's law

Isotropy = properties are independent of direction (contrary to anisotropy)

$$\begin{Bmatrix} \frac{1}{E}, -\frac{\mu}{E}, -\frac{\mu}{E}, 0,0,0 \\ -\frac{\mu}{E}, \frac{1}{E}, -\frac{\mu}{E}, 0,0,0 \\ -\frac{\mu}{E}, -\frac{\mu}{E}, \frac{1}{E}, 0,0,0 \\ 0,0,0, \frac{1}{G}, 0,0 \\ 0,0,0,0, \frac{1}{G}, 0 \\ 0,0,0,0,0, \frac{1}{G} \end{Bmatrix}$$

Polycrystalline materials – metals, ceramics, polymers, partly also

…. amorphous materials – glass

only two independent characteristics

$$G = \frac{E}{2(1+\mu)}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_x + \sigma_z)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

Additional elastic characteristics, only two of them are independent

$\sigma = E\varepsilon$ $\tau = G\gamma$	**standard form of Hooke's law for isotropic material loaded in tension and shear**
$p = -K\dfrac{\Delta V}{V}$	modulus of elasticity of volume
μ	Poisson ratio

Important equations and quantities

Additional elastic characteristics, only two of them are independent

$$E = 2G\left(1 + \mu\right)$$

$$K = \frac{E}{3\left(1 - 2\mu\right)}$$

$$\mu = \frac{E}{2G} - 1$$

!

Important equations and quantities

Additional elastic characteristics, only two of them are independent

Isotropic material: μ a E

Poisson's ratio μ – ratio of relative transverse reduction in area to relative longitudinal elongation in field of elastic deformations.

The value μ represents elastic compressibility of the body, i.e. ability of the material to decrease (at compression) and to increase (at tension) its volume in the course of elastic deformation.

Definition of the μ

relative transverse reduction in area

to

relative longitudinal elongation

$$\mu = \left| \frac{\Delta d / d_0}{\Delta l / l_0} \right| = -\frac{\Delta d / d_0}{\Delta l / l_0} = -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{\varepsilon_3}{\varepsilon_1}$$

$$\varepsilon_3 = \varepsilon_2 = -\mu\varepsilon_1$$

Definition of the μ

What is the value of the μ ?

$$(1+\Delta V) = (1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3) =$$

$$(1+\varepsilon_1)(1-\mu\varepsilon_1)(1-\mu\varepsilon_1) =$$

$$\left\{\begin{array}{l}\text{neglecting the members in} \\ \text{second and third power } \varepsilon^2, \varepsilon^3\end{array}\right\}$$

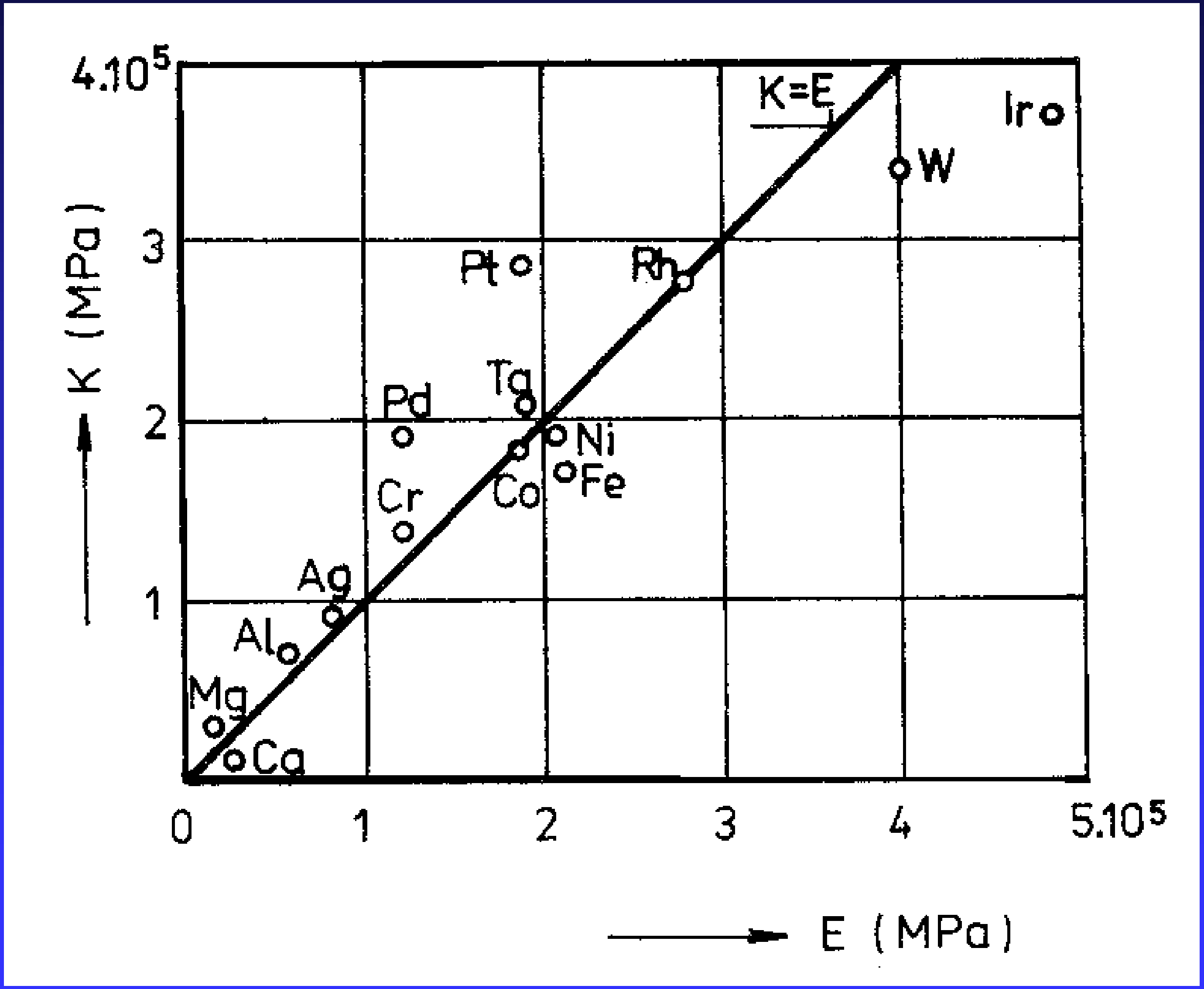
$$= 1+\varepsilon_1(1-2\mu)$$

$$\text{the volume increases} \Rightarrow\Rightarrow 1-2\mu > 0 \Rightarrow\Rightarrow \mu < 0.5$$

$$\text{the volume is constant } \mu = 0.5$$

Relation between K and E

For metals, *K* is equal to *E*



What is the value of μ for metals ?

$$E \approx K$$

$$1 = 3(1 - 2\mu)$$

$$\mu = 0.33$$

$$G = \frac{3}{8}E$$

$$K = \frac{E}{3(1-2\mu)}$$

$$E = 2*10^5 \text{ MPa} = 200 \text{ GPa}$$

$$G = 7.5*10^4 \text{ MPa} = 75 \text{ GPa}$$

Elastic characteristics - measurements

- Quasi-static methods
- Dynamic methods

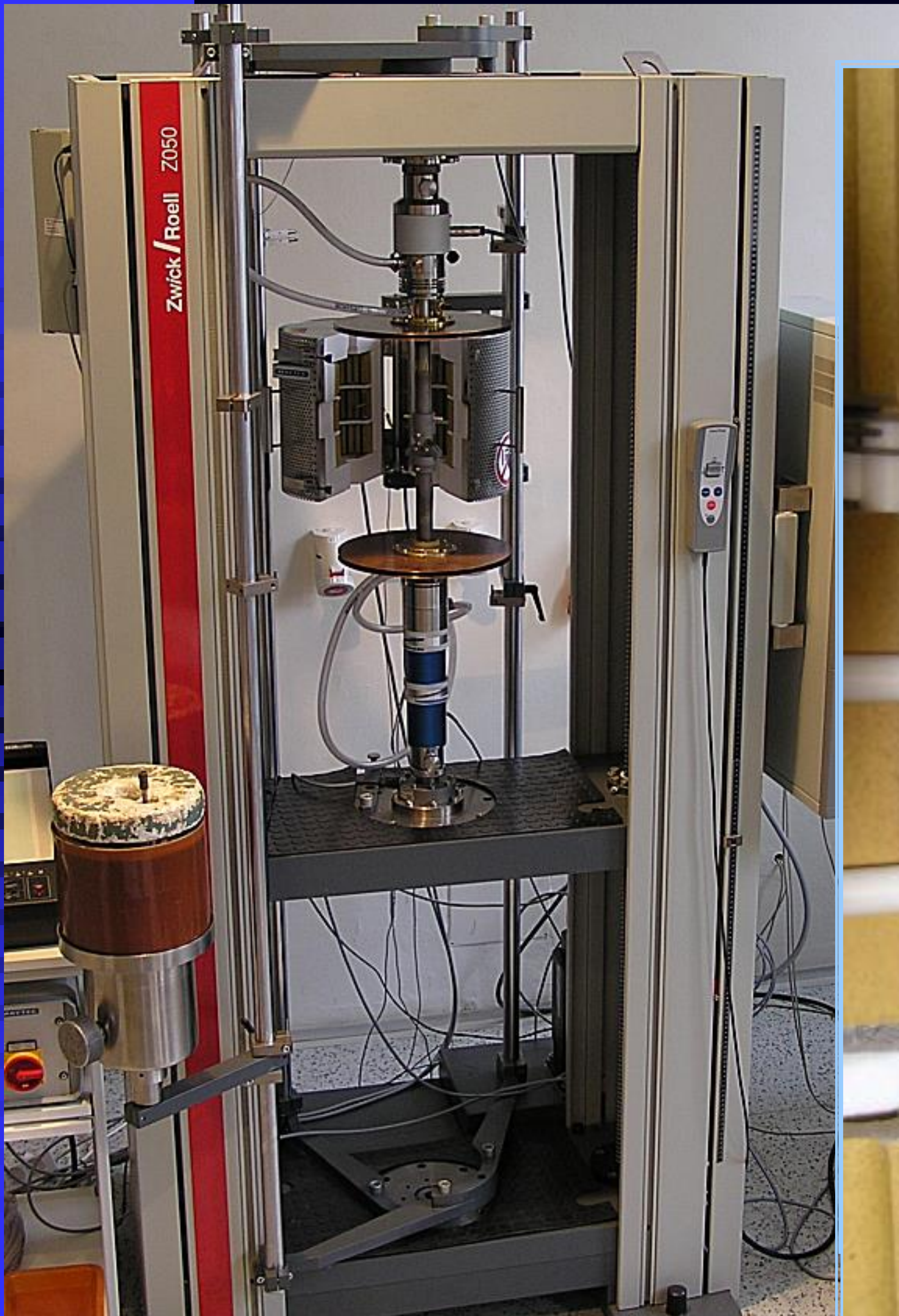
low frequency

high frequency

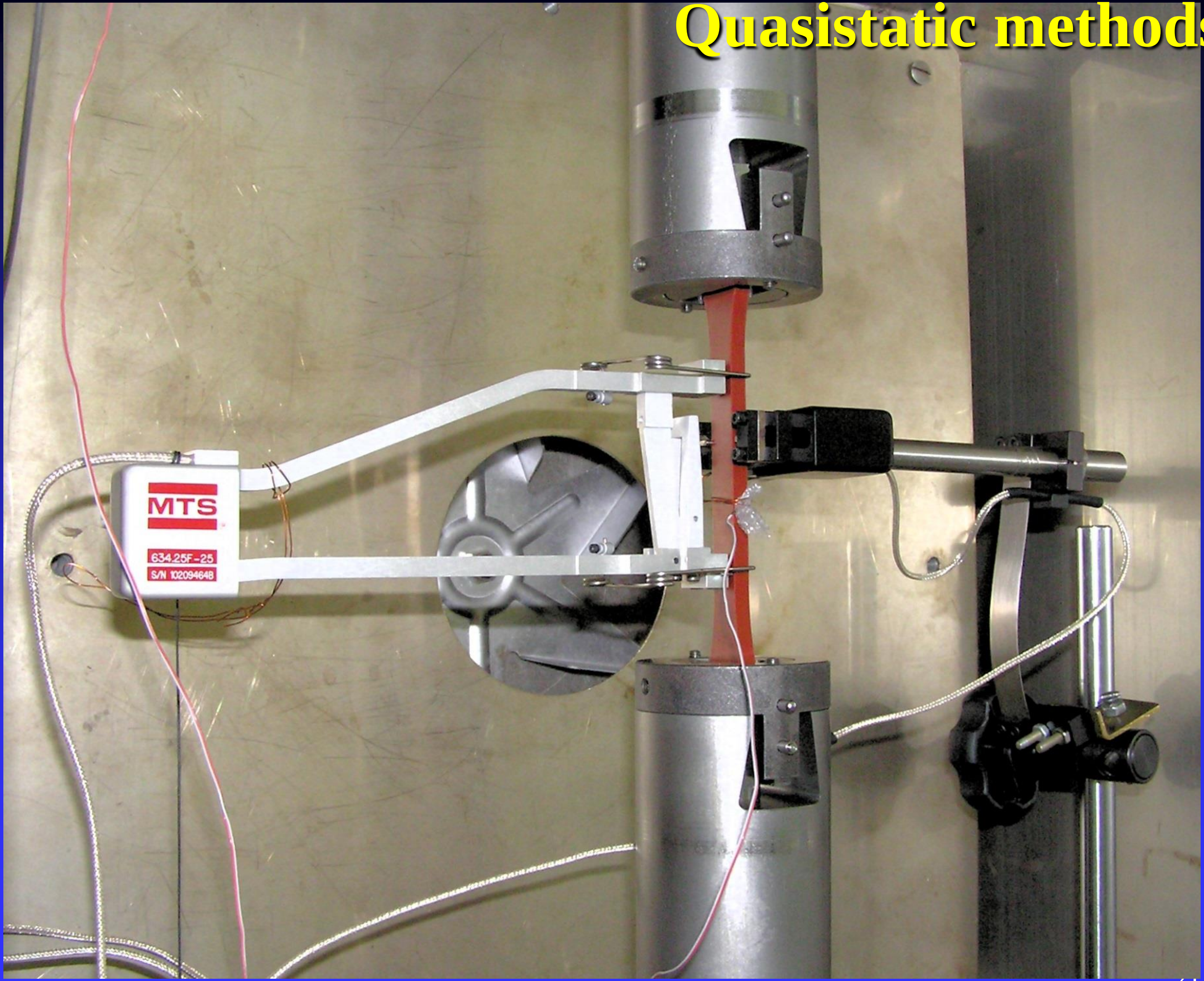
Quasistatic methods

- Poisson ratio – from definition; using tensometers and/or transducers of longitudinal elongation and transversal contraction
- Modulus E – tensile test
  – flexural test
- Modulus G – torsion test

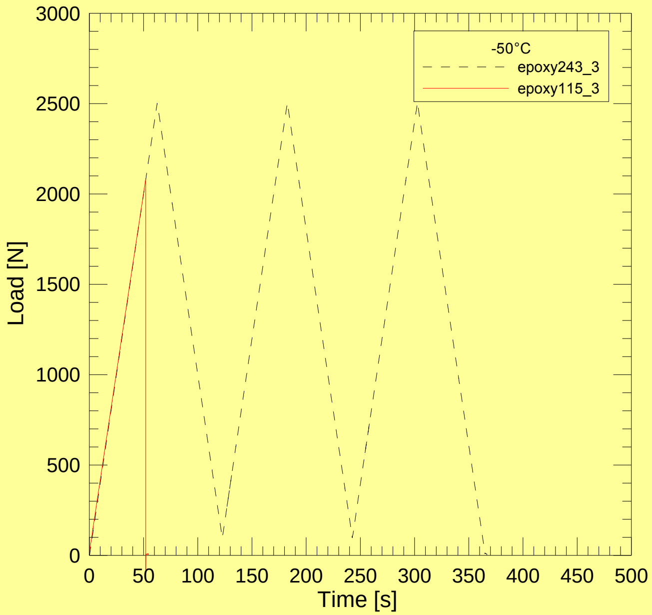
Quasistatic methods

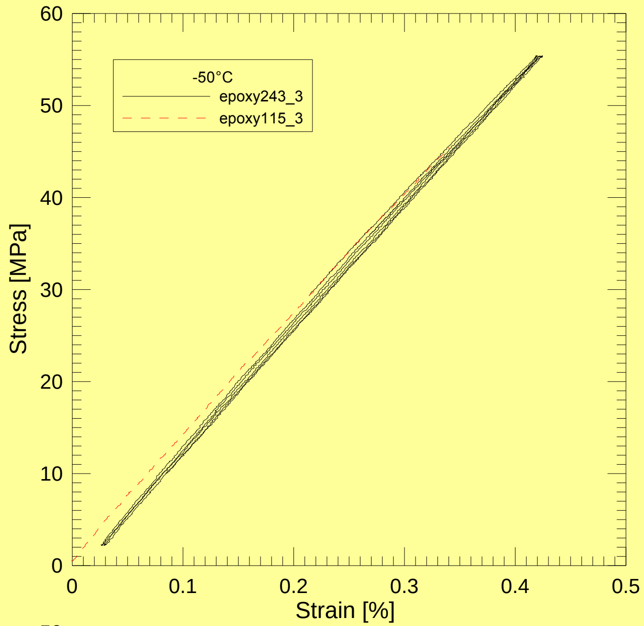


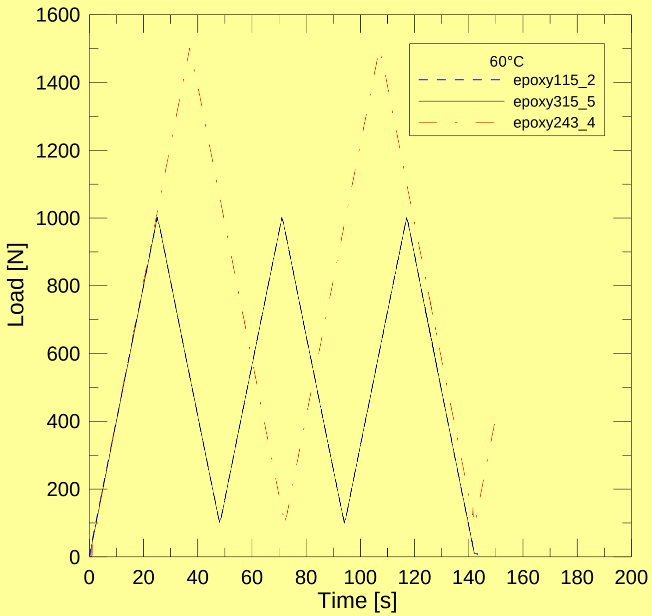
Quasistatic methods

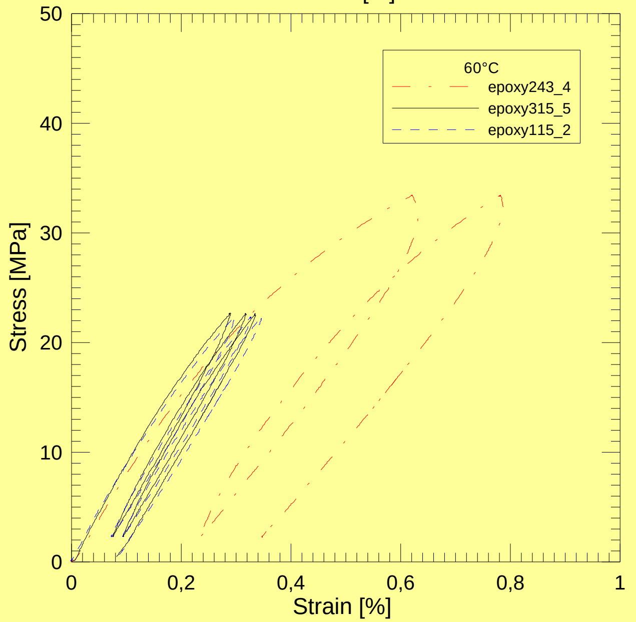


Quasistatic methods



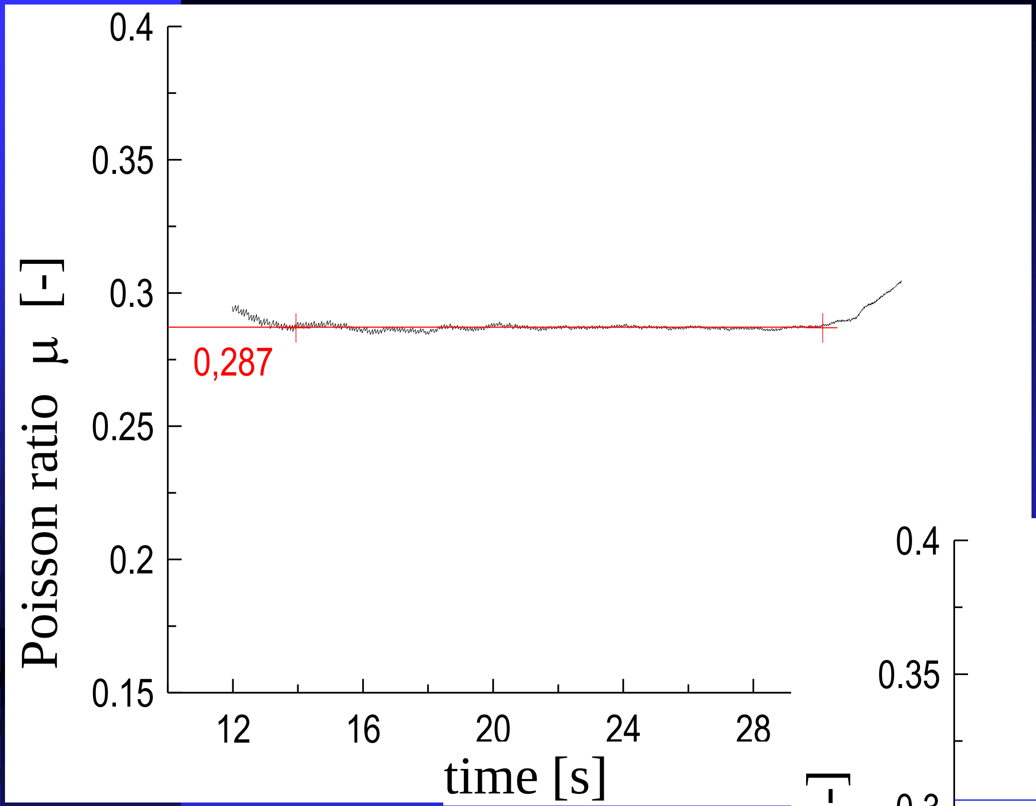


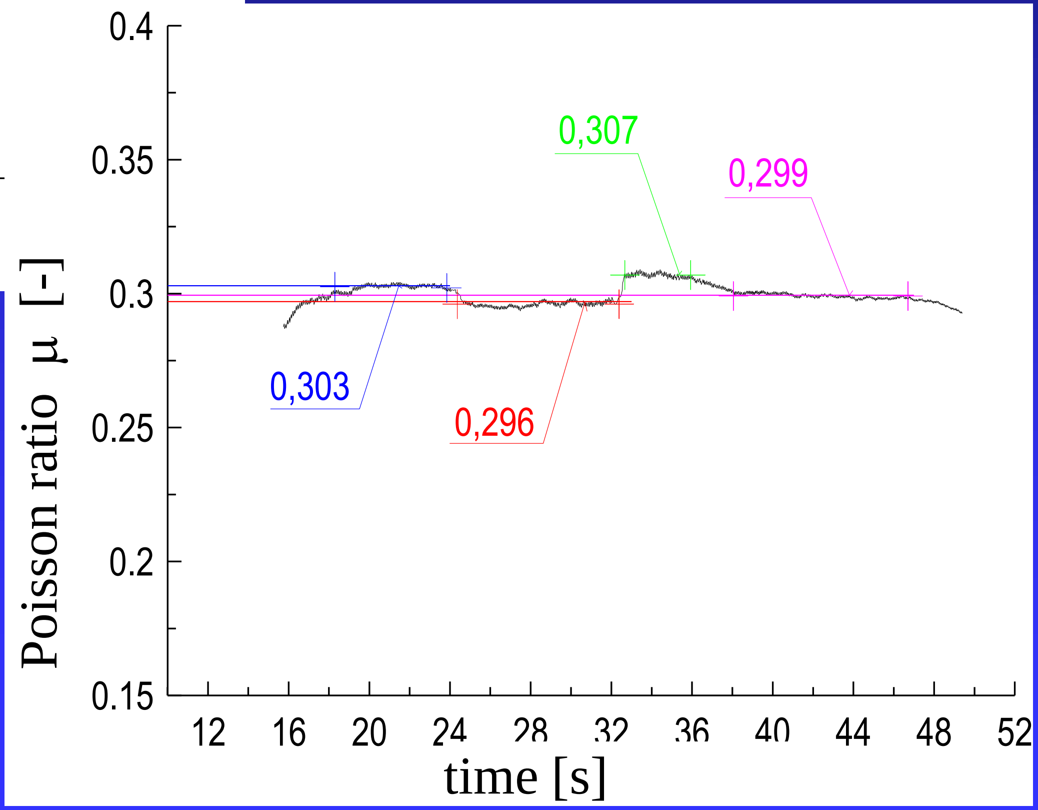




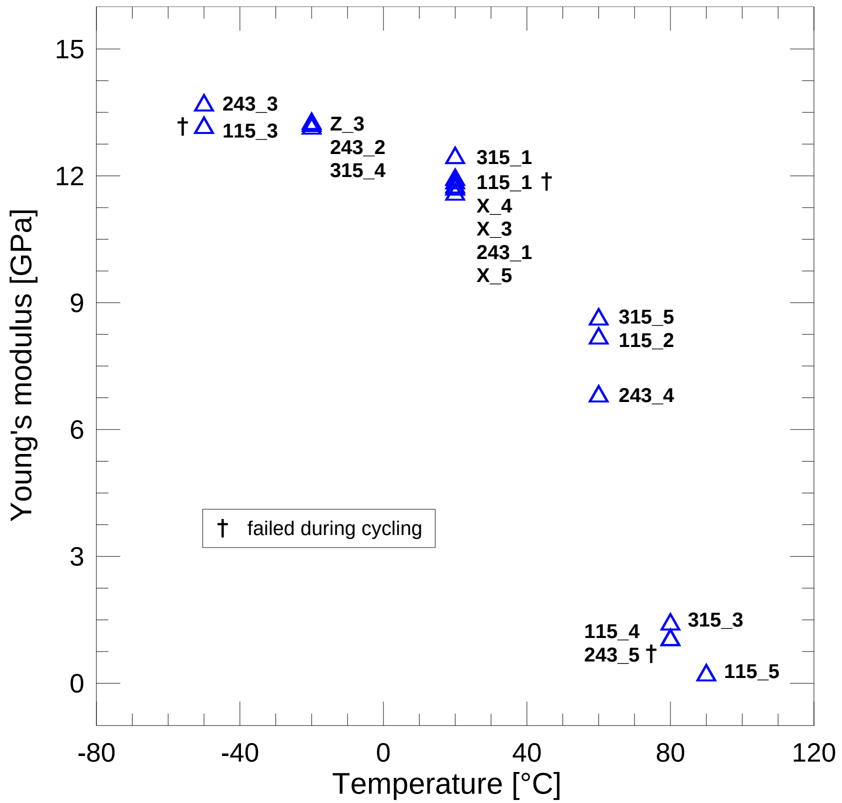
Quasistatic methods

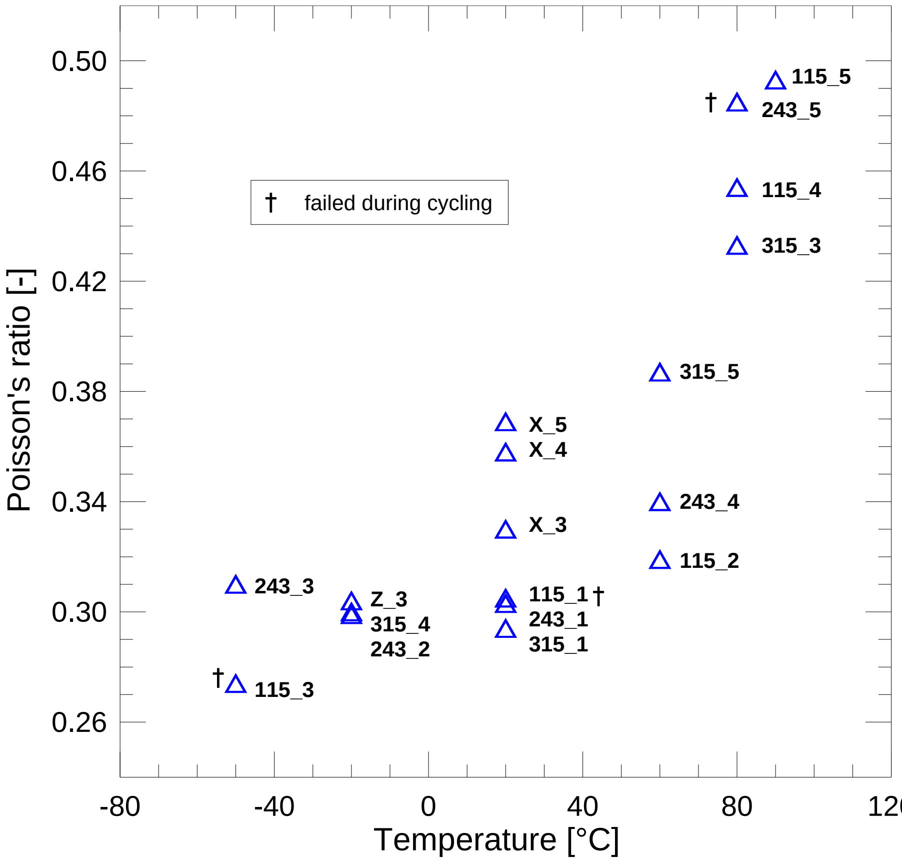
Poisson's ratio





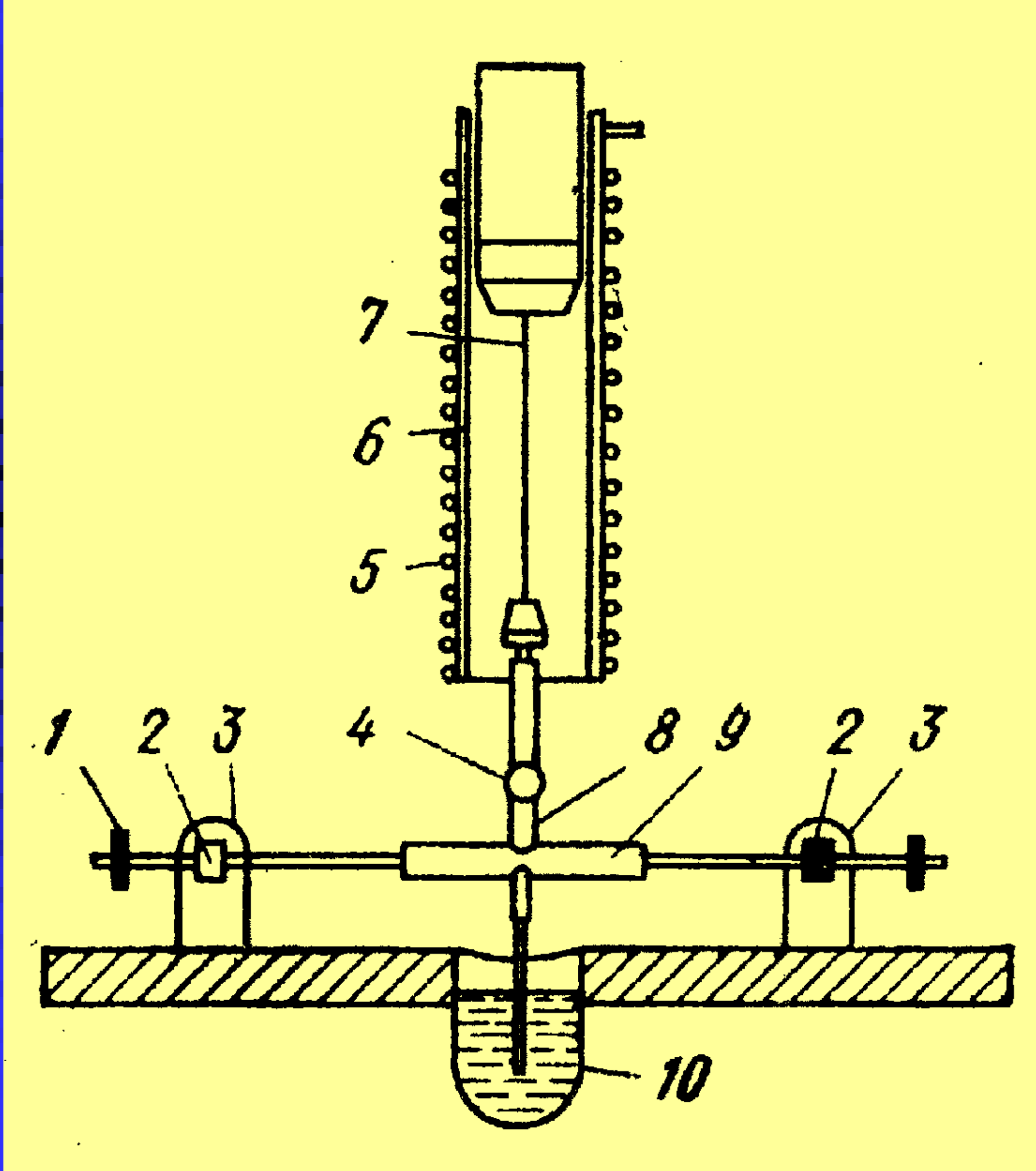
Quasistatic methods





Dynamic methods

torsional pendulum



$$G = 128\pi . J_a . l . f^2 / d^4$$

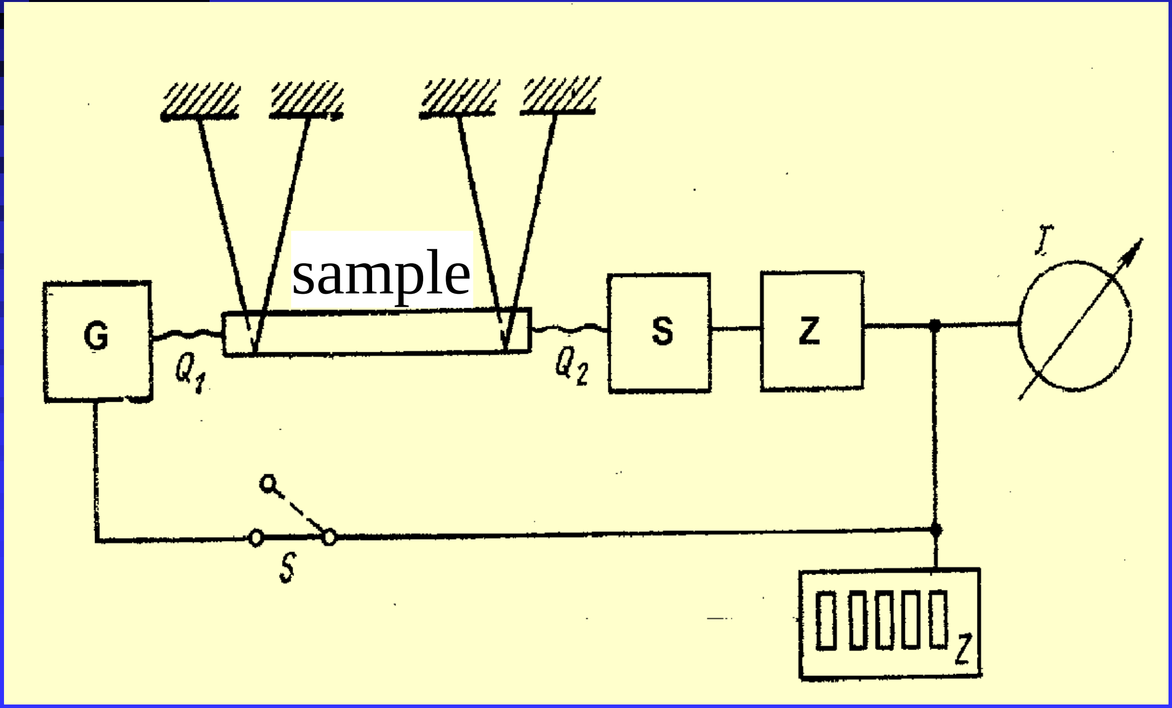
- d – sample diameter
- l – sample length
- J_a – axial moment of inertia
- f – eigenfrequency of pendulum vibrations

Dynamic methods
- high frequency

$v_D = (E/\rho)^{1/2}$ Speed of longitudinal wave propagation

for sample of length l_0
eigenfrequency of vibrations f, and
n harmonic wave

$$v_{Dn} = 2l_0 f_n/n, \ (n = 2,3,4\ldots)$$



$$E = 4.l_0^2.\rho.f_n^2/n^2$$

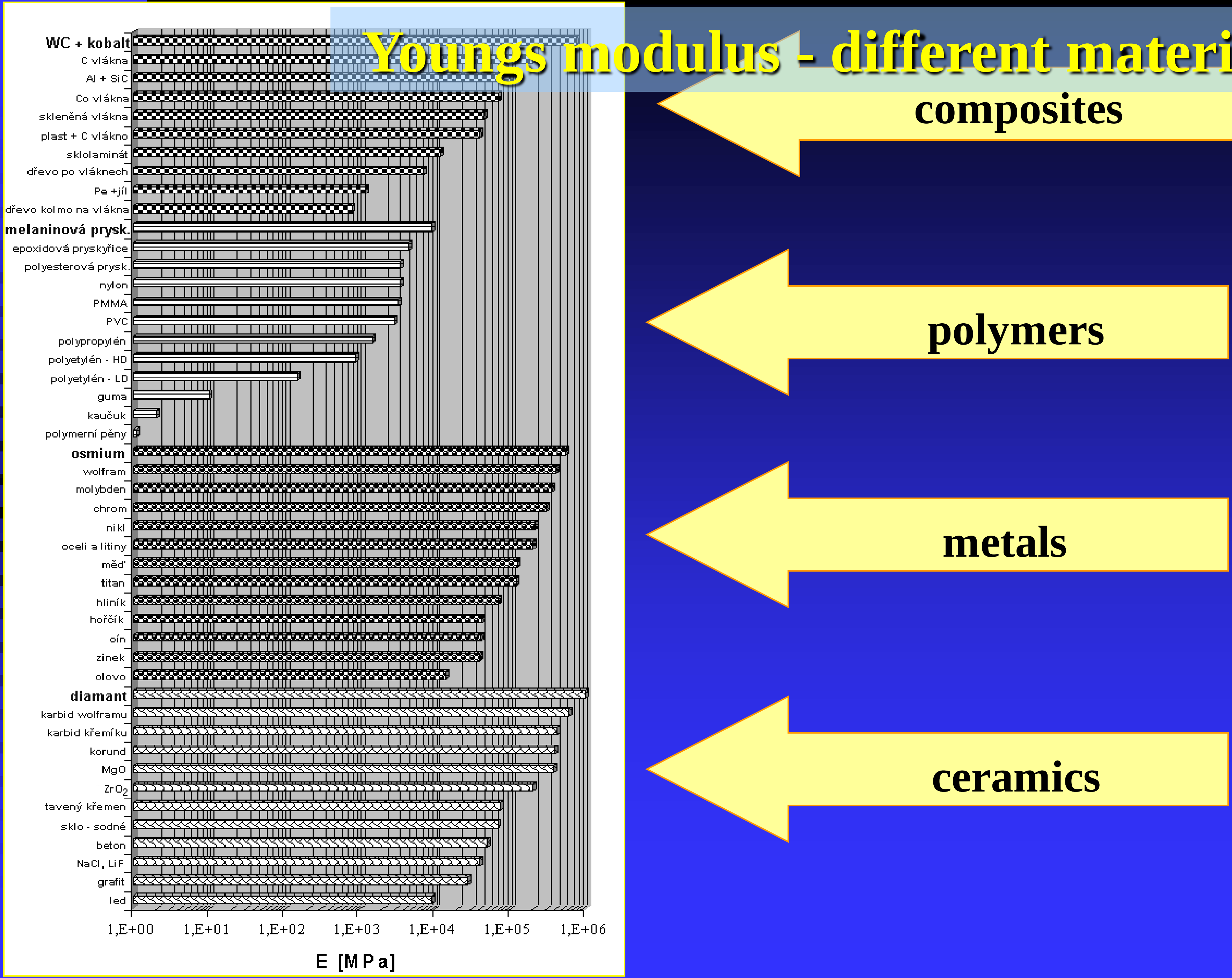
Grindosonic

Vibrations introduced by small hammer blow (metallic, wood, plastic ..)

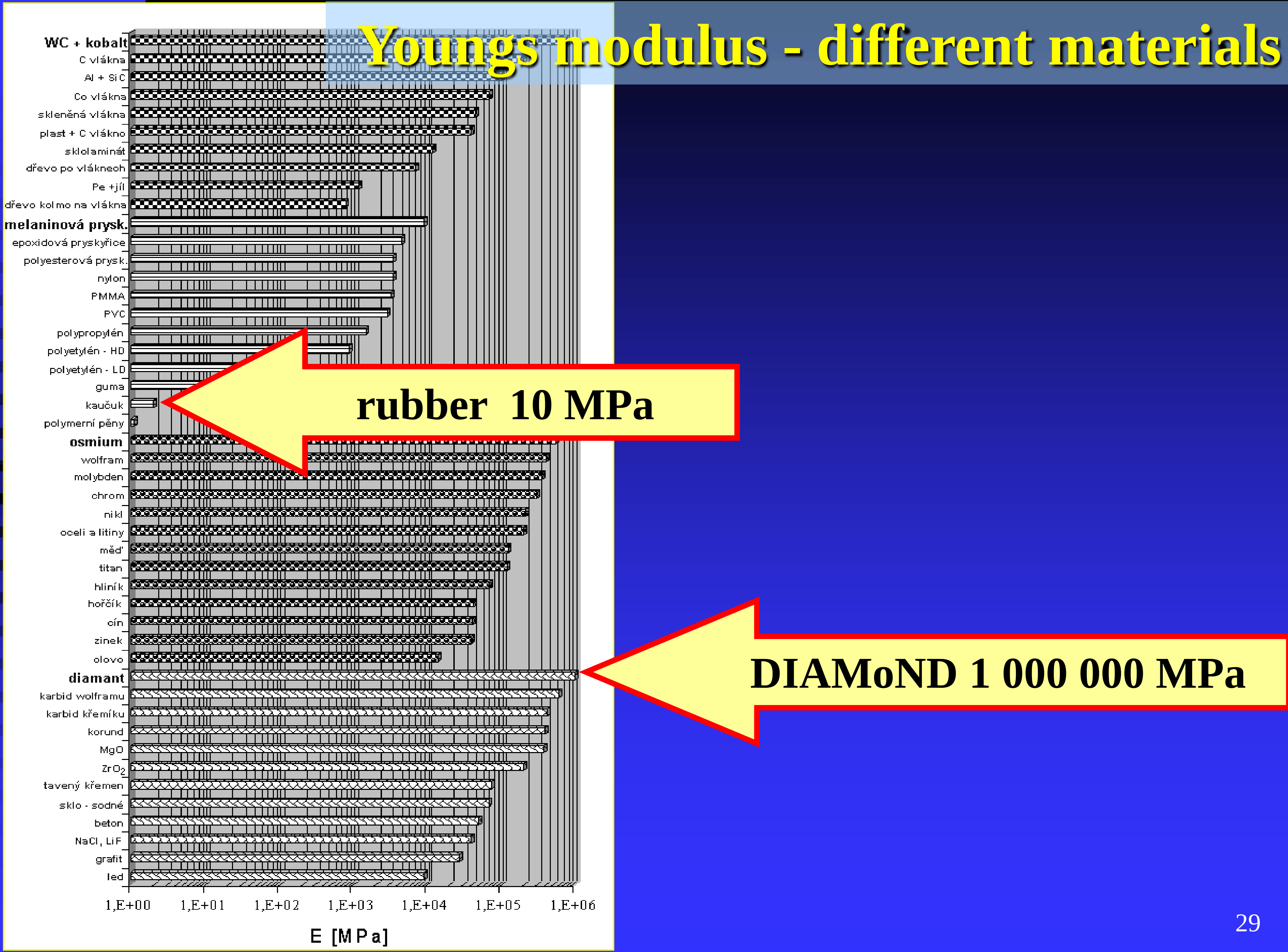
Elastic properties measurements

- differences between E modulus values obtained at quasistatic and dynamic loading, (about 10% for metals, ceramics; for polymers of several orders)
- when measuring elastic properties for applications where vibrations are important, dynamic methods of measurements are prefered

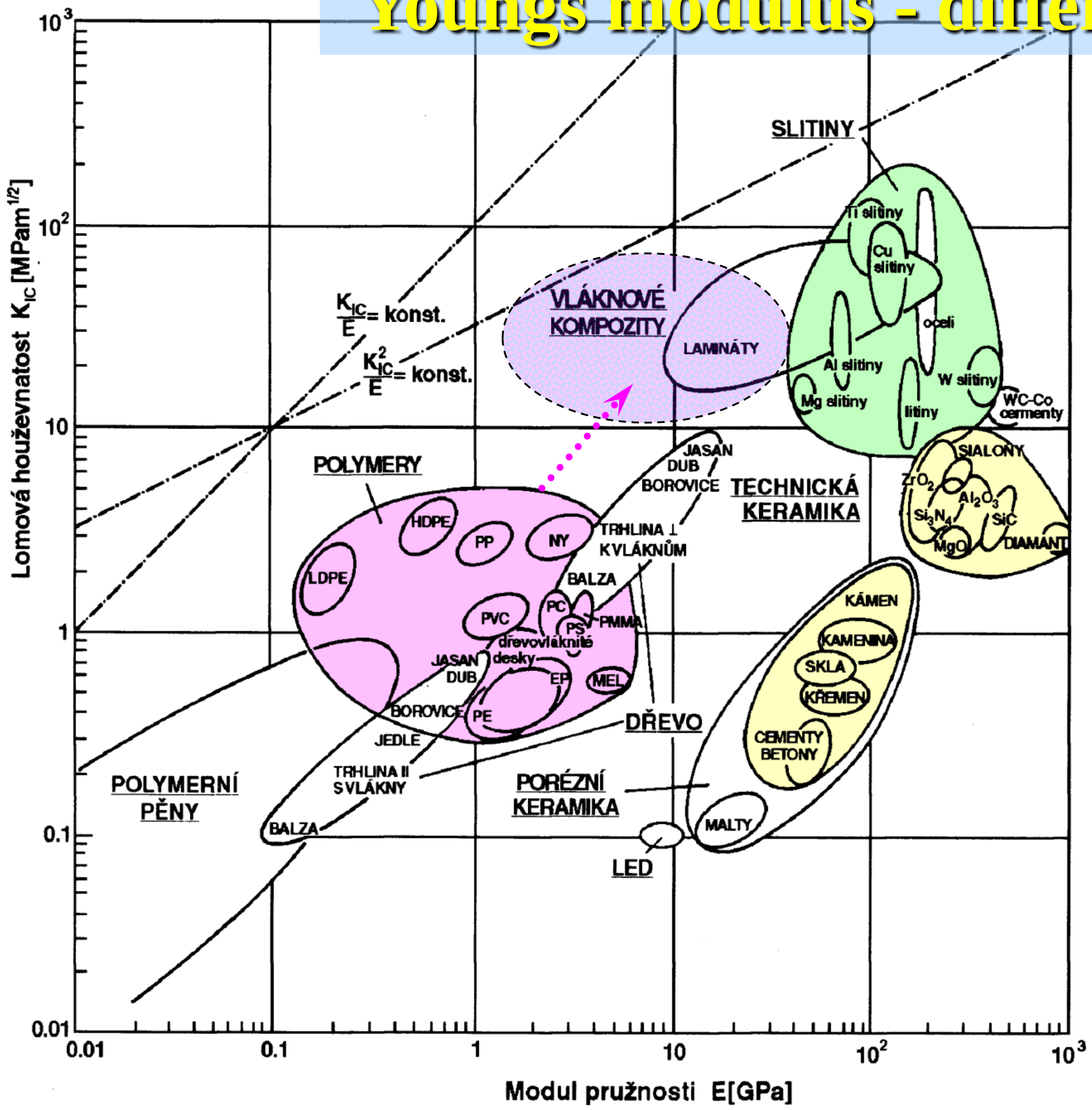
Youngs modulus - different materials



Youngs modulus - different materials



Youngs modulus - different materials



Young's modulus - different materials

$$\text{metals, ceramics, glass} \quad \left(10^4 \text{ to } 10^6\right) \text{MPa}$$

$$\text{polymers} \left(1 \text{ to } 10^4\right) \text{MPa}$$

elastic deformation: dependence of Young's modulus and rigidity on

- interatomic bonding

- atoms ordering in space

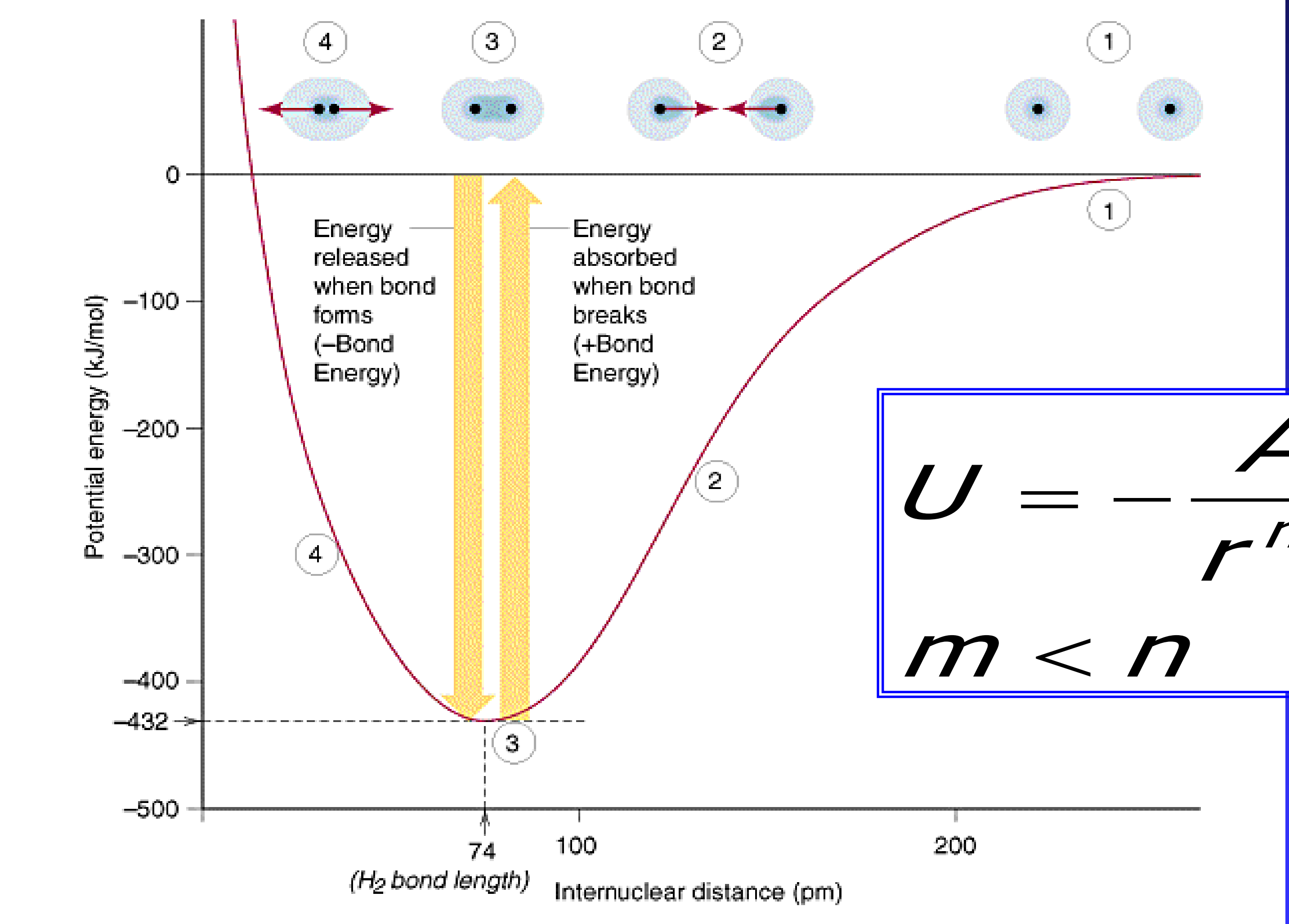
Elasticity

- ✓ Young's modulus
- 2) Interatomic bonds
- 3) Atoms ordering in solids
- 4) Physical basis of Young's modulus

Interatomic bonds

Interatomic bonds – primary (T_m 1000K to 5000K)

– secondary (T_m 100K to 500K)



$$U = -\frac{A}{r^m} + \frac{B}{r^n}$$

$$m < n$$

Interatomic bonds

Thanks to interatomic bonds condensed state of matter

Metallic bond - ions of metal atoms in cloud of electrons
>> close packed ordering = highest density

Ion bond - more hevily filled crystallic lattice – octaedric and tetraedric positions filled

Covalent bond – complicated lattice, highest density, highest strength

Interatomic bonds

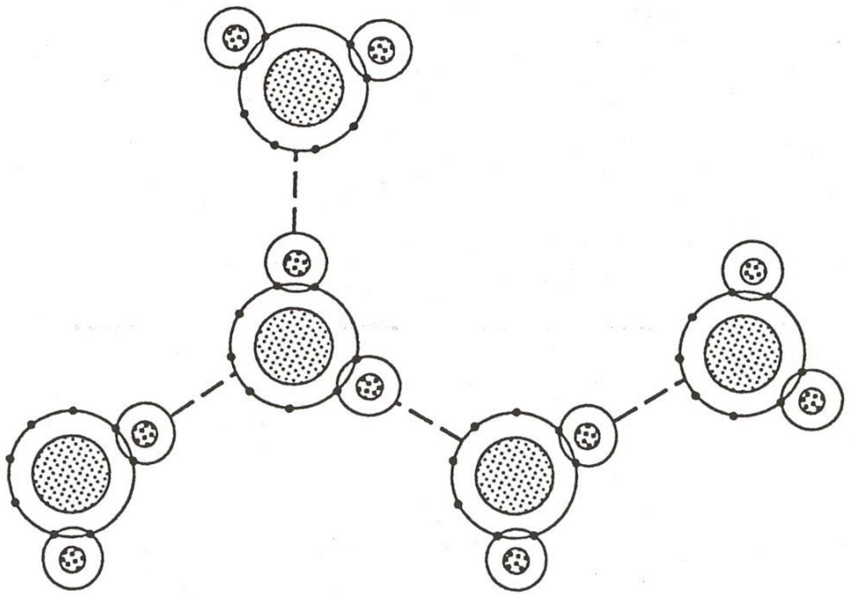
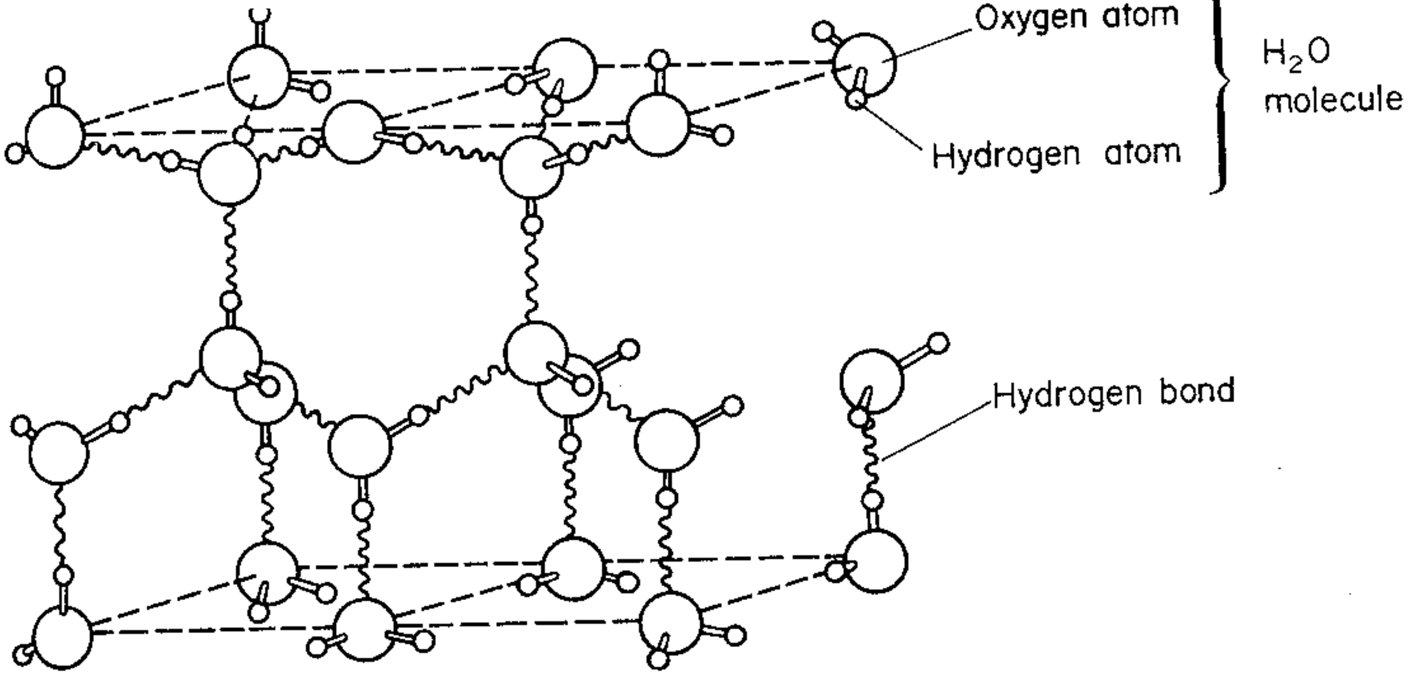


Figure 2.7 Oxygen-to-hydrogen secondary bonds between water (H_2O) molecules.

Matter performance controlled by <u>secondary</u> bonds



Interatomic bonds

Matter performance controlled by primary bonds

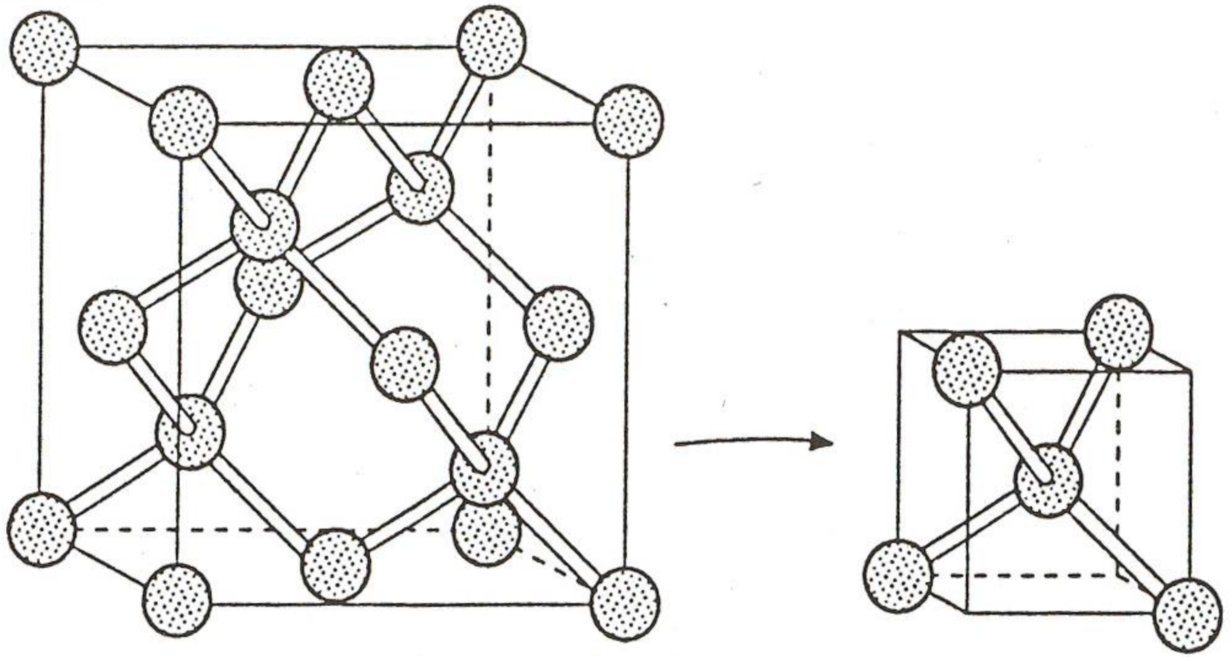


Figure 2.5 Diamond cubic crystal structure of carbon. As a result of the strong and directional covalent bonds, diamond has the highest melting temperature, the highest hardness, and the highest elastic modulus E, of all known solids.

condensed states

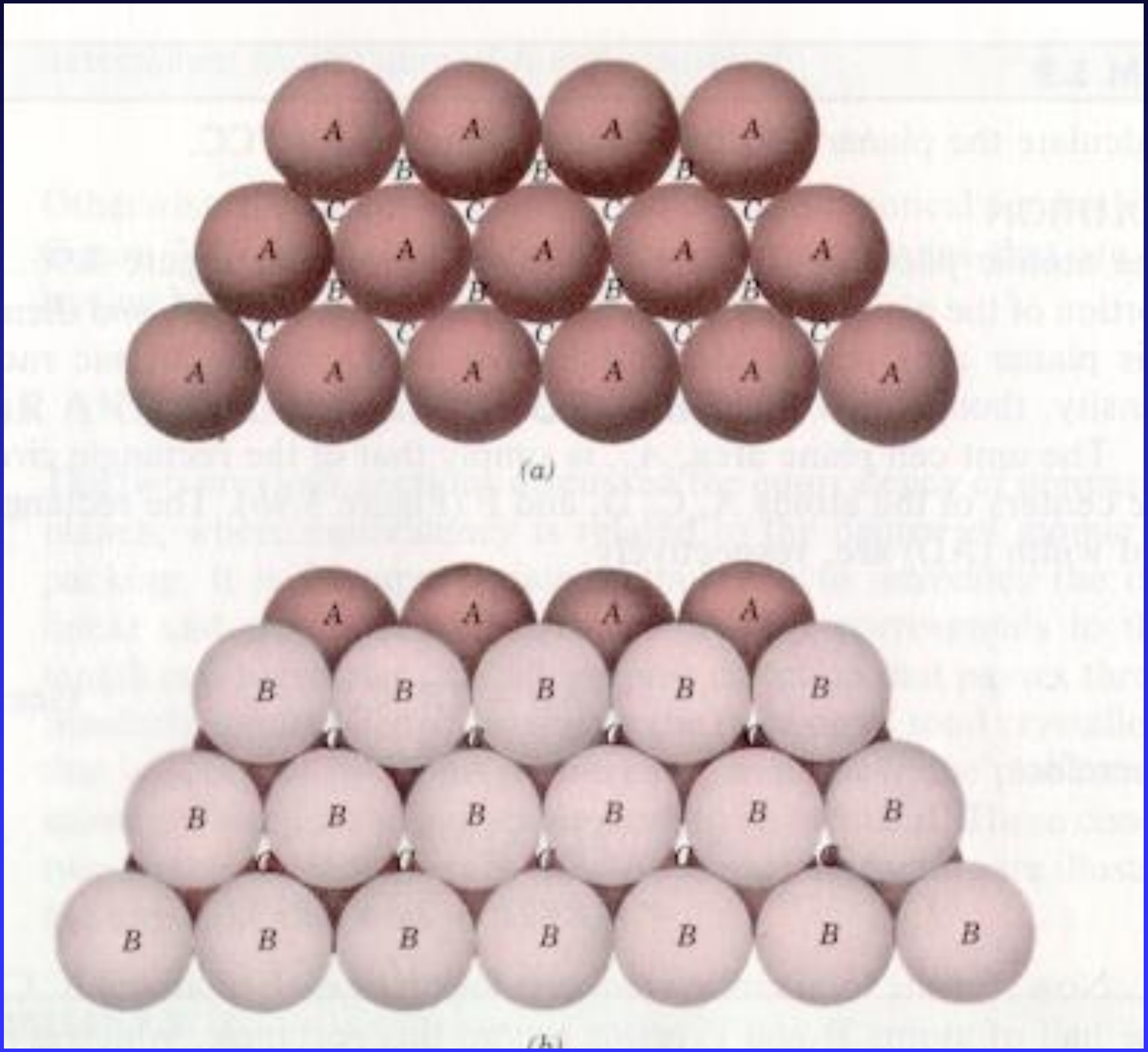
state		bonds		K	E
		disordered	strong		
1	liquid	*		big	zero
2	Liquid crystal	*		big	small ≠ 0
3	Rubber	Secondary	Primary	big	E<<K
4	Glass		*	big	E ≈ K
5	Crystal		*	big	E ≈ K

Elasticity

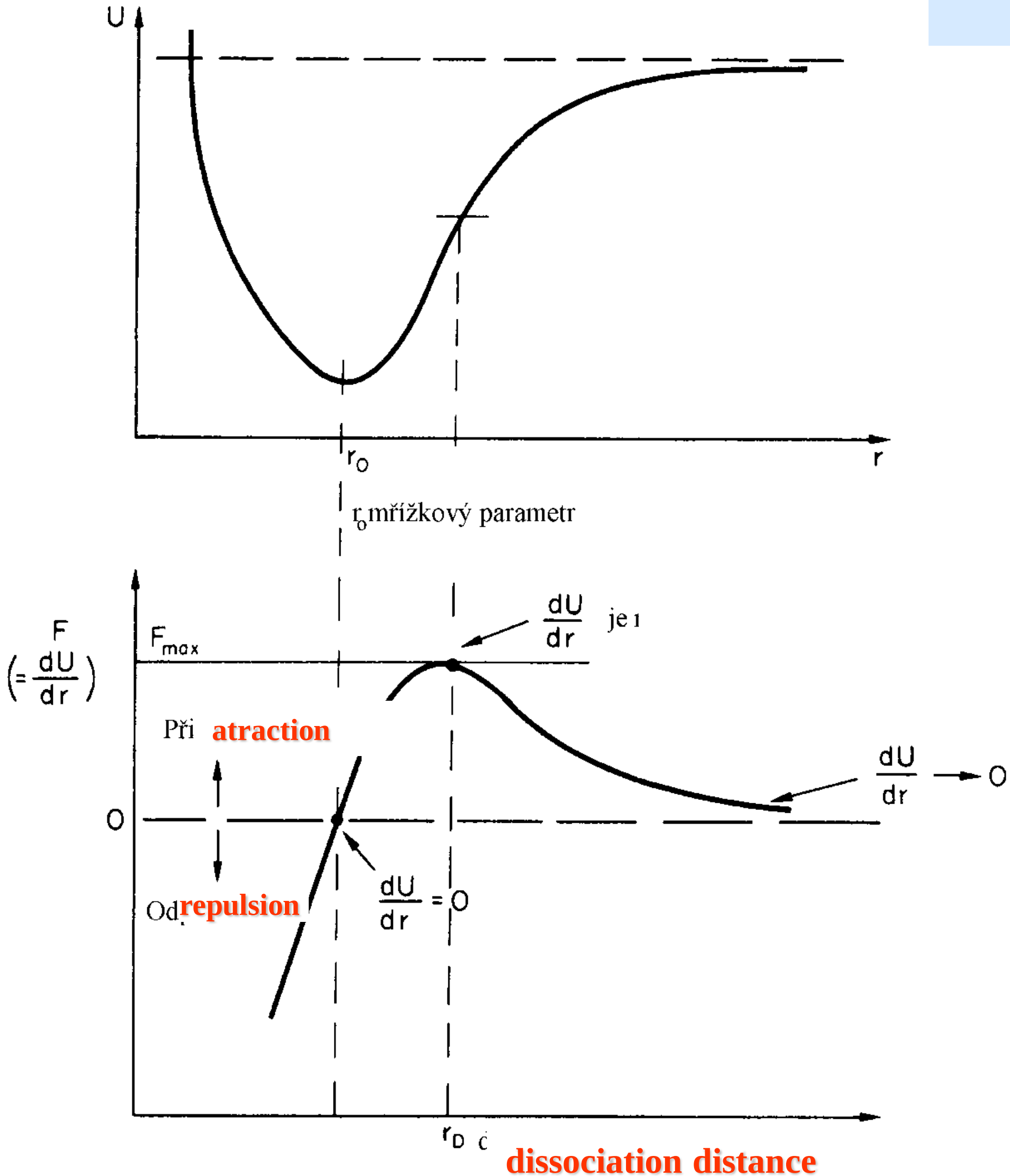
- ✓ Young’s modulus
- ✓ Interatomic bonds

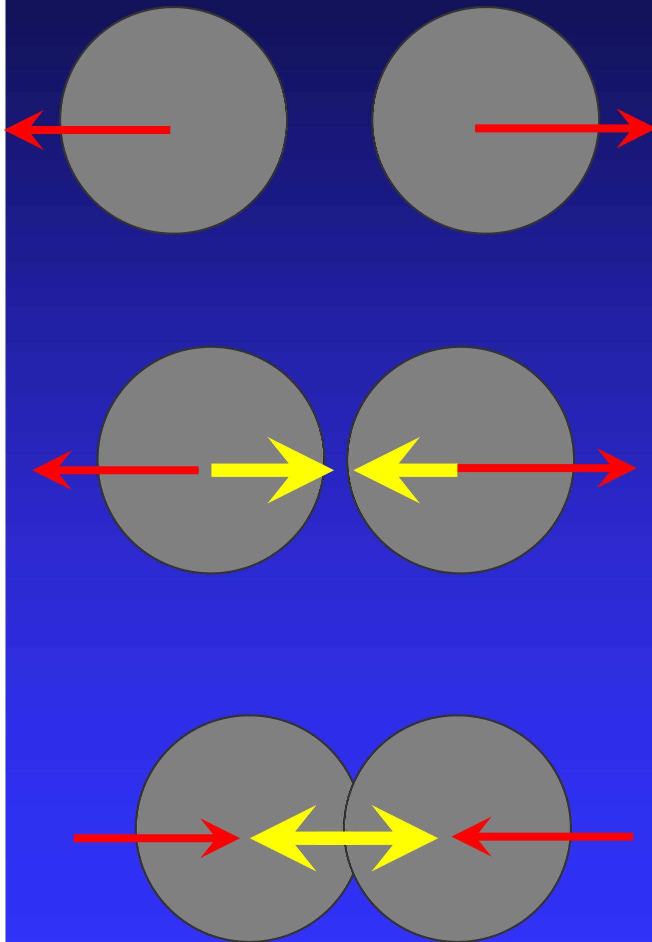
2) Atoms ordering in solids
3) Physical basis of Young’s modulus

Close packed lattice - metals

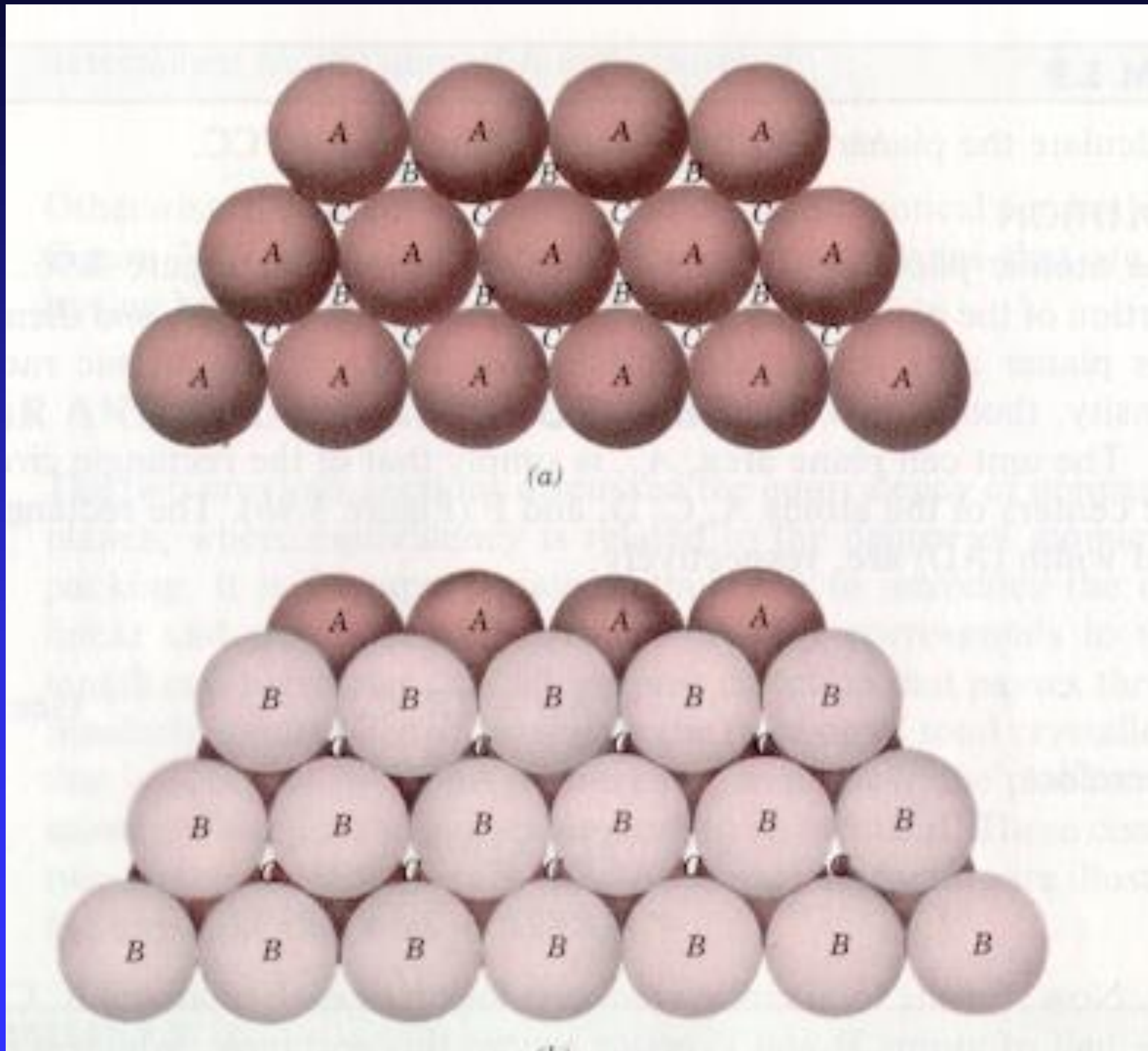


Atoms ordering

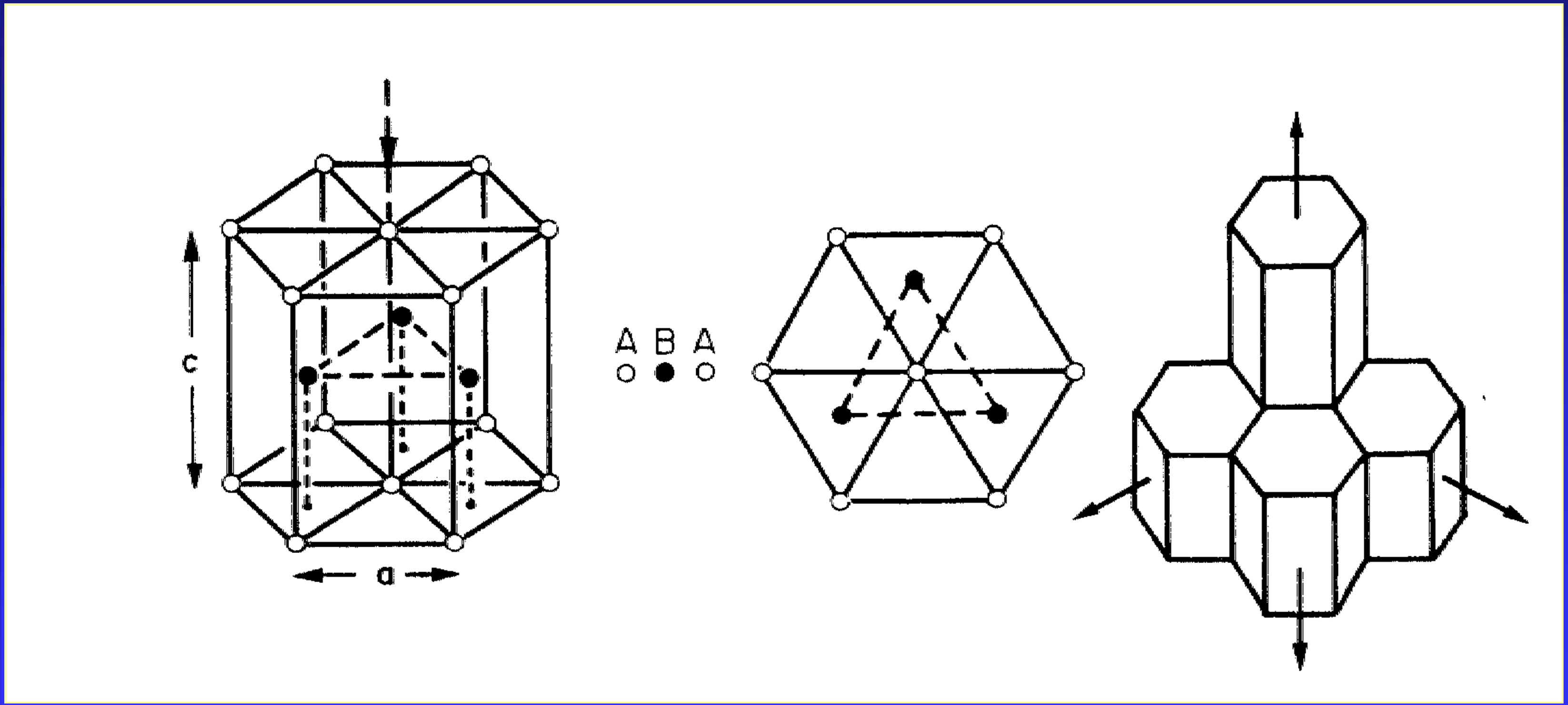




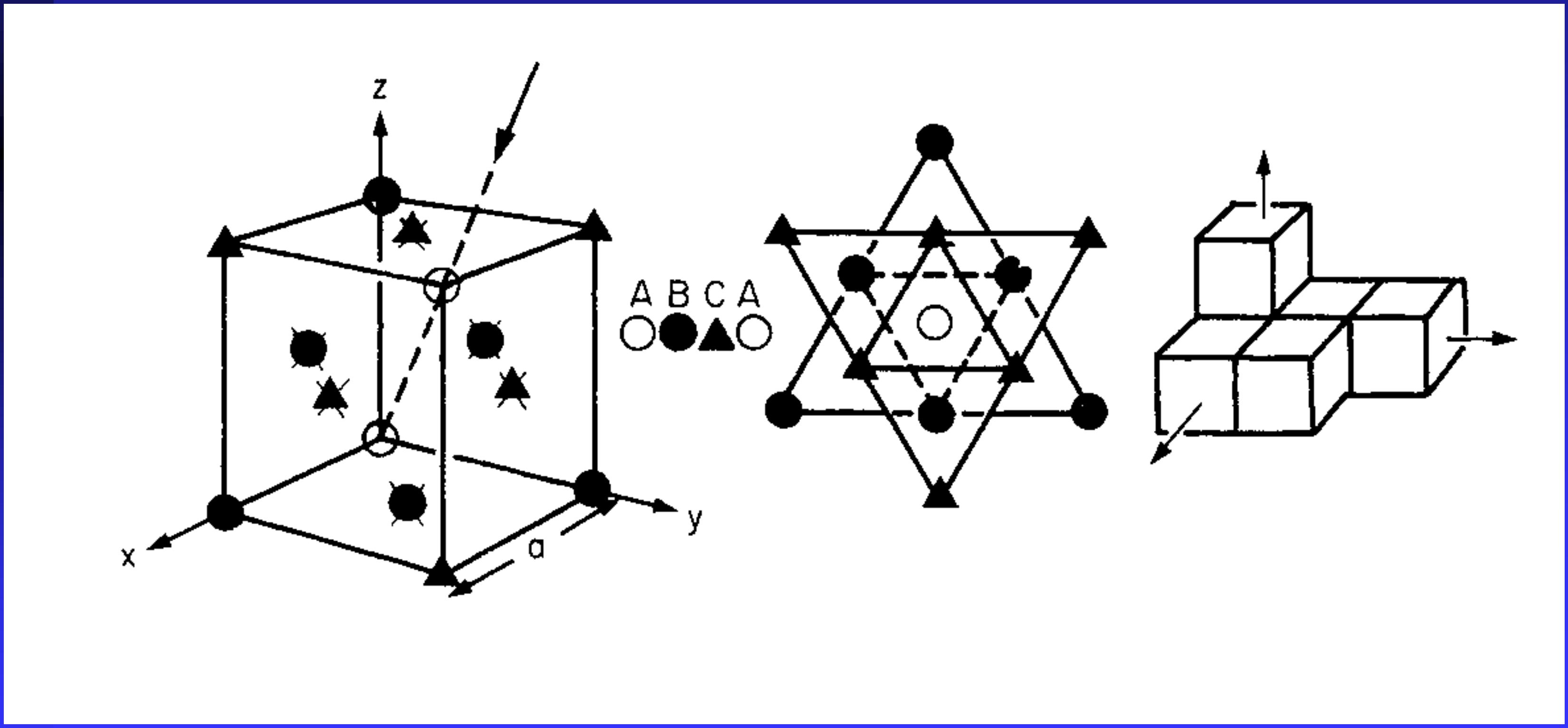
Close packed lattice - metals



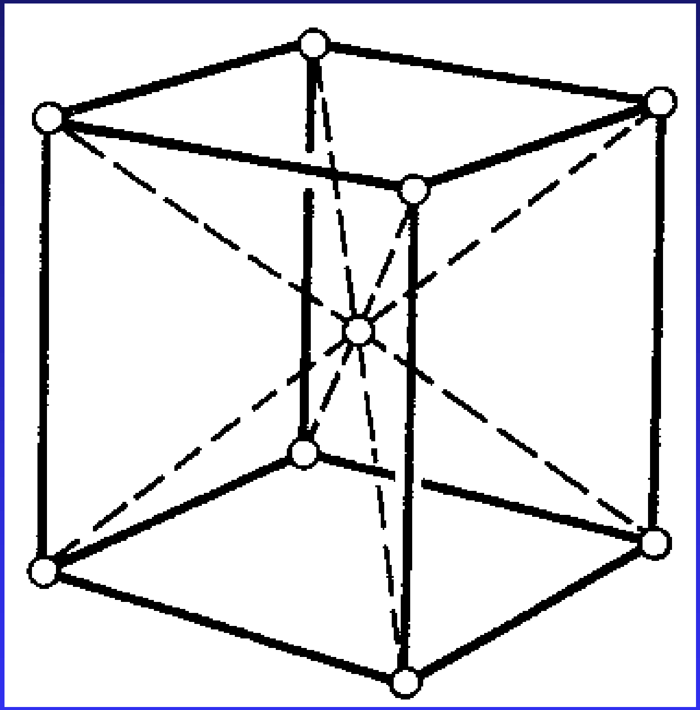
Structure hcp –Mg, Zn, Co, a - Ti hexagonal - close – packed structure



Structure fcc – Al, Cu, Au, Ag, Pt, Ni, γ-Fe
face – centred – cubic structure

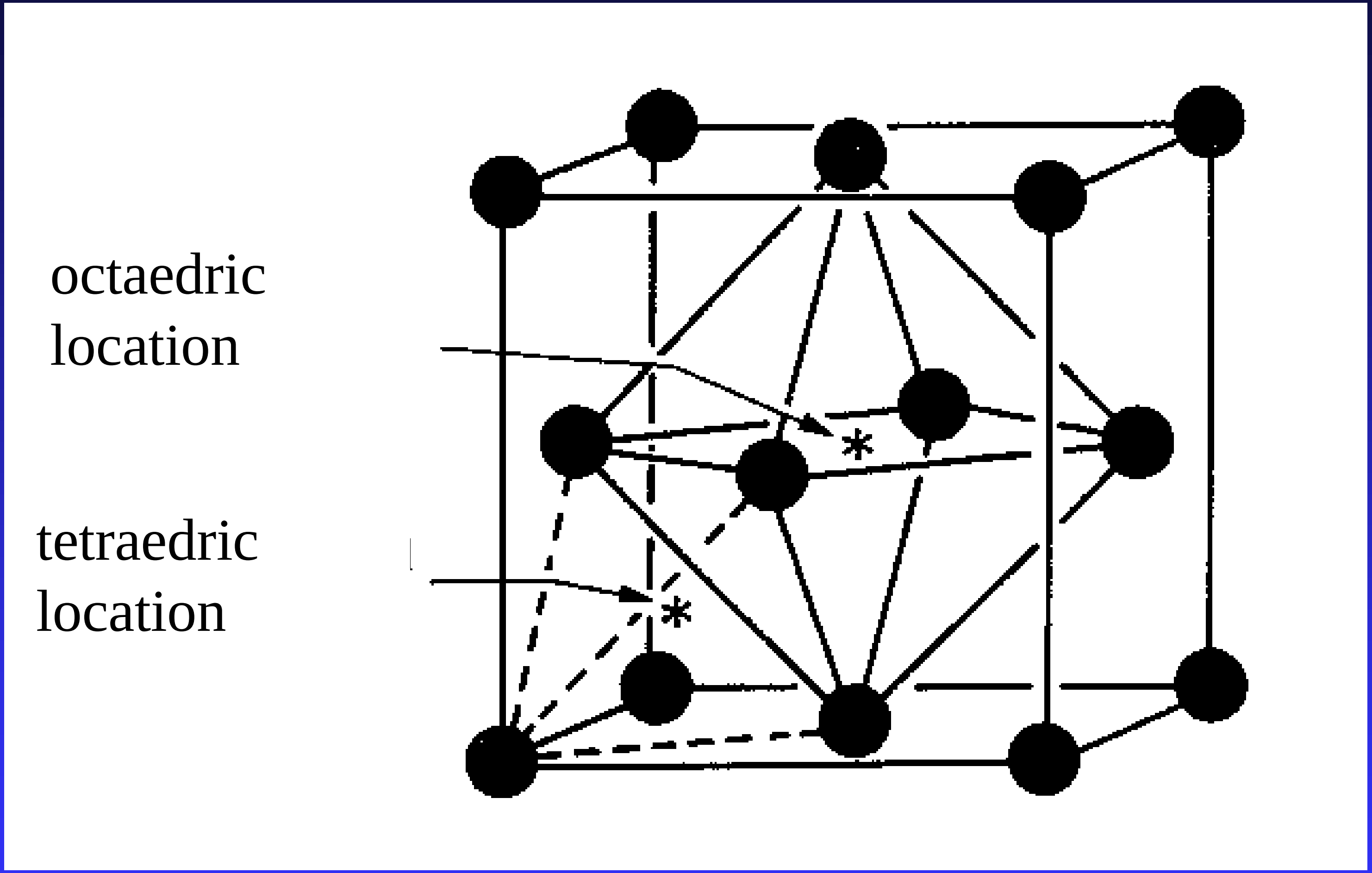


Structure bcc – α-Fe, Mo, W, Ta
body – centred - cubic structure

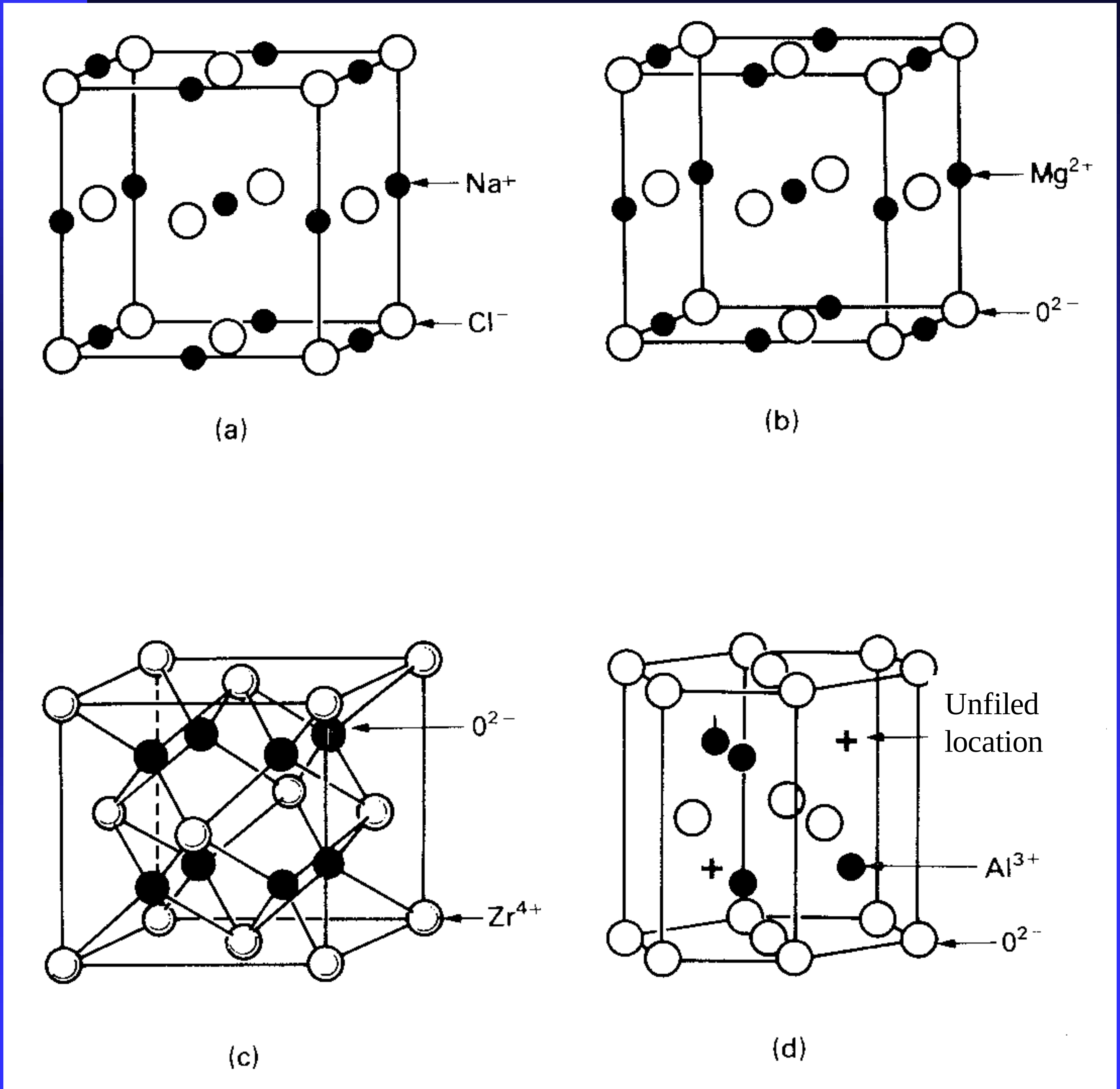


- Polymorphy – transformation of lattice with temperature, $\alpha - \gamma - \delta$ (910/1400)°C

Ion bonding - ceramics



Ion bonding - ceramics



Covalent bonding - ceramics

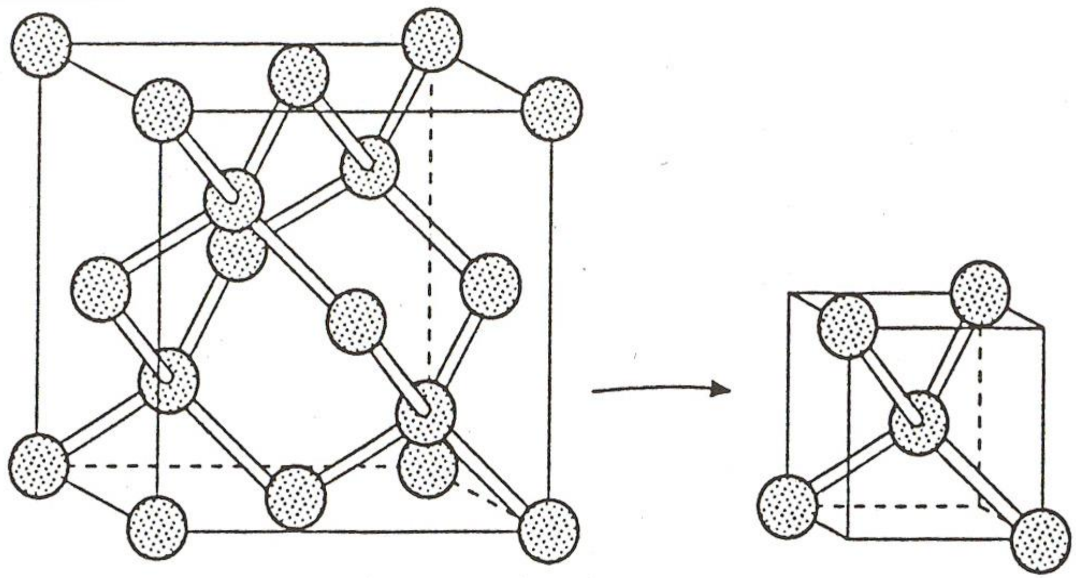
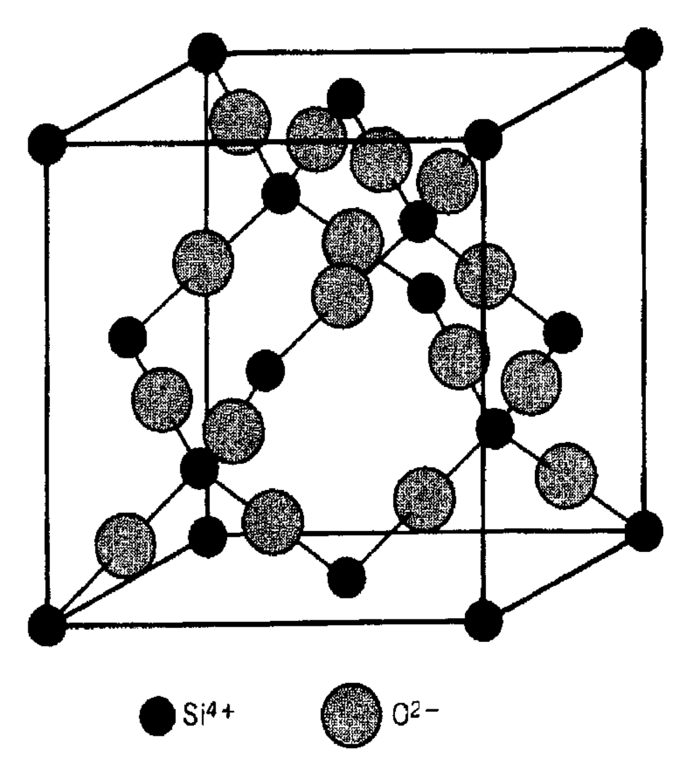


Figure 2.5 Diamond cubic crystal structure of carbon. As a result of the strong and directional covalent bonds, diamond has the highest melting temperature, the highest hardness, and the highest elastic modulus E, of all known solids.

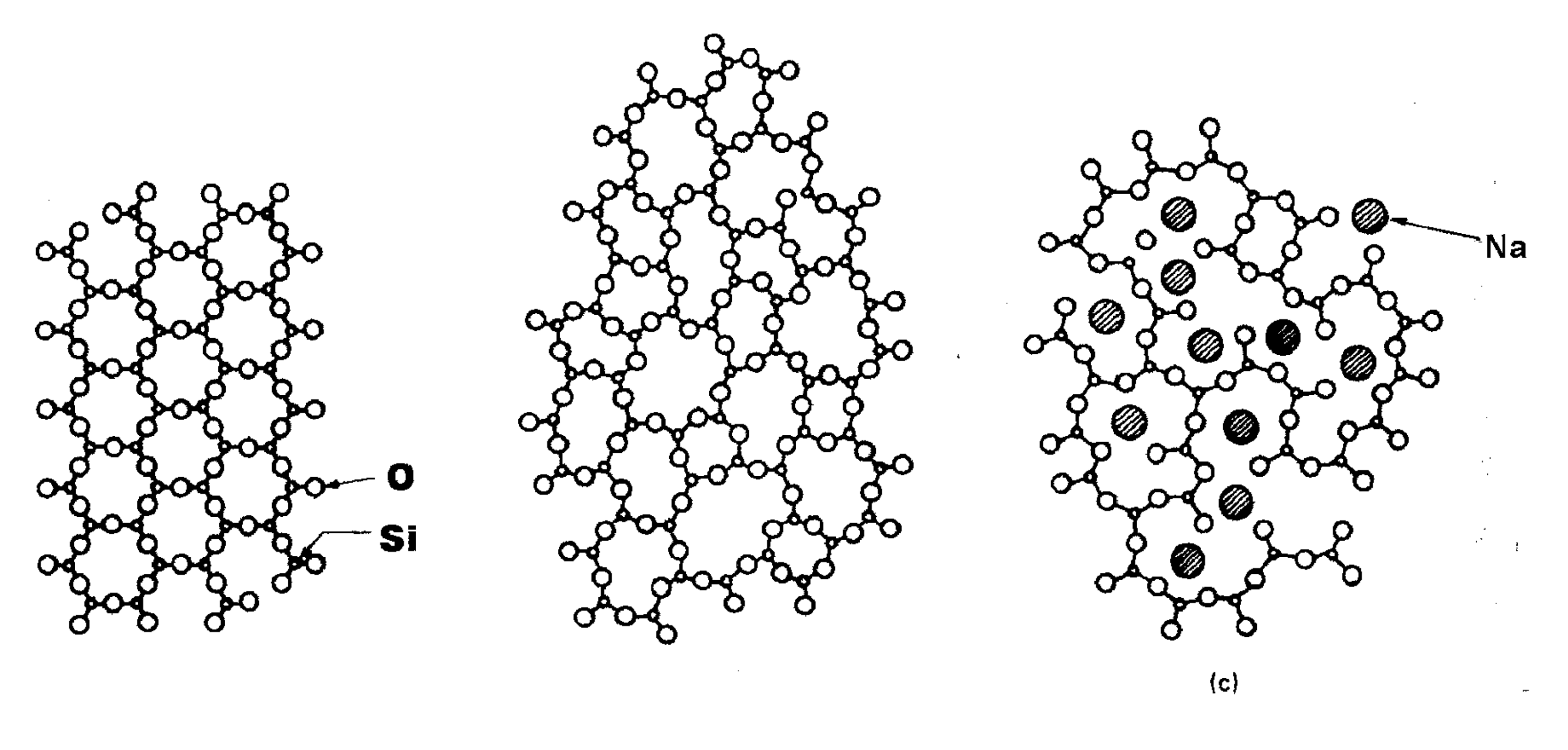
Diamond



Silica

Lattice is not exactly the close packed, but atoms fully filled space

Covalent bonding - glass



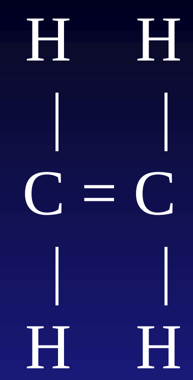
Silica glass – melting point 1200°C

Na, Ca, Fe – terminators– 700°C

Covalent bonding – polymers (plastics)

- Termoplastic PE, PP, PS, PMMA
- Termosets (reactoplasts)
- Elastomers (rubbers, elastic)

Etylen



monomer

Number of monomers (n)	Temperature of softening	State at 20°C
1	-167	gas
6	-12	liquid
35	37	oil
140	93	wax
430	109	solids

Polymerization degree $10^3 \approx 10^5$

Polymers (plastics)

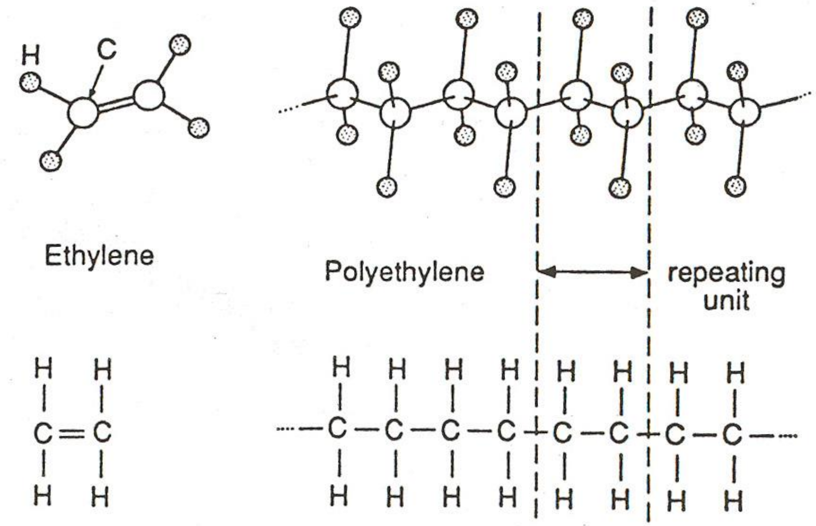


Figure 2.6 Molecular structures of ethylene gas (C_2H_4) and polyethylene polymer. The double bond in ethylene is replaced by two single bonds in polyethylene, permitting formation of the chain molecule.

```
H    H    H    H    H    H    H    H
|    |    |    |    |    |    |    |
C -  C -  C -  C -  C -  C -  C -  C
|    |    |    |    |    |    |    |
H    R    H    R    H    R    H    R
```

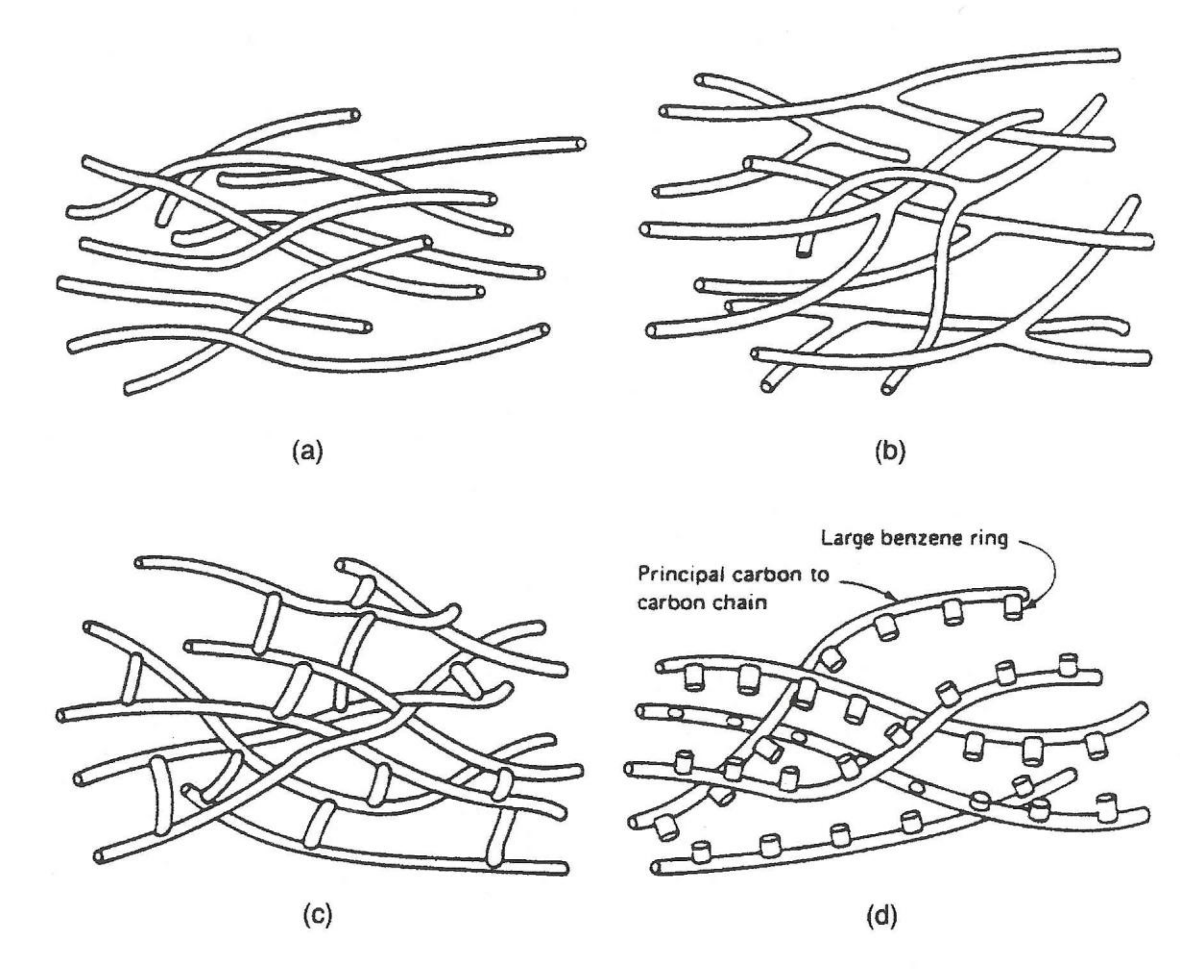
R ---- H **PE / polyethylene** (tubes, isolators)

R ---- CH_3 **polypropylen** (better resistance against light)

R ---- Cl **polyvinylchlorid** (roofing and floor covering)

R ---- C_6H_5 **polystyrén** (gramophone disks, cutlery, příbory, izolation)

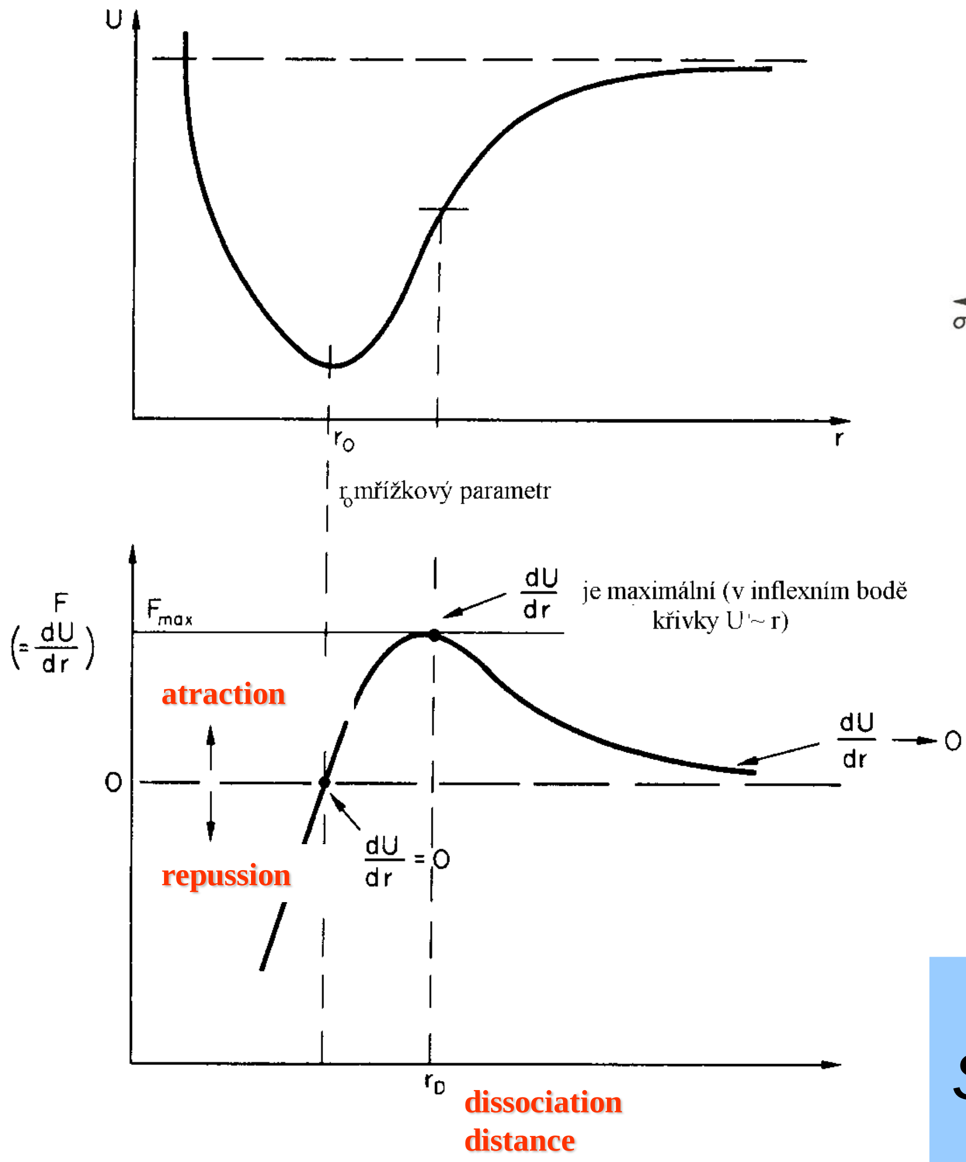
Shape of plastics molecule

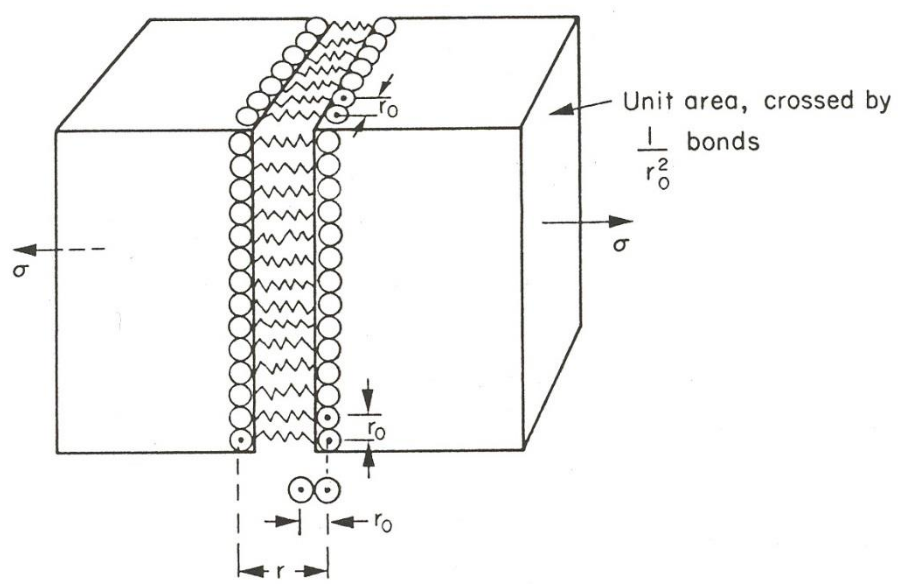


Elasticity

- ✓ Young's modulus
- ✓ Interatomic bonds
- ✓ Atoms ordering in solids
- Physical basis of Young's modulus

Physical nature of elastic modulus





rigidity of the bond

(second order derivative of the potential energy)

$$S_0 = \left(\frac{d^2U}{dr^2} \right)_{r=r_0}$$

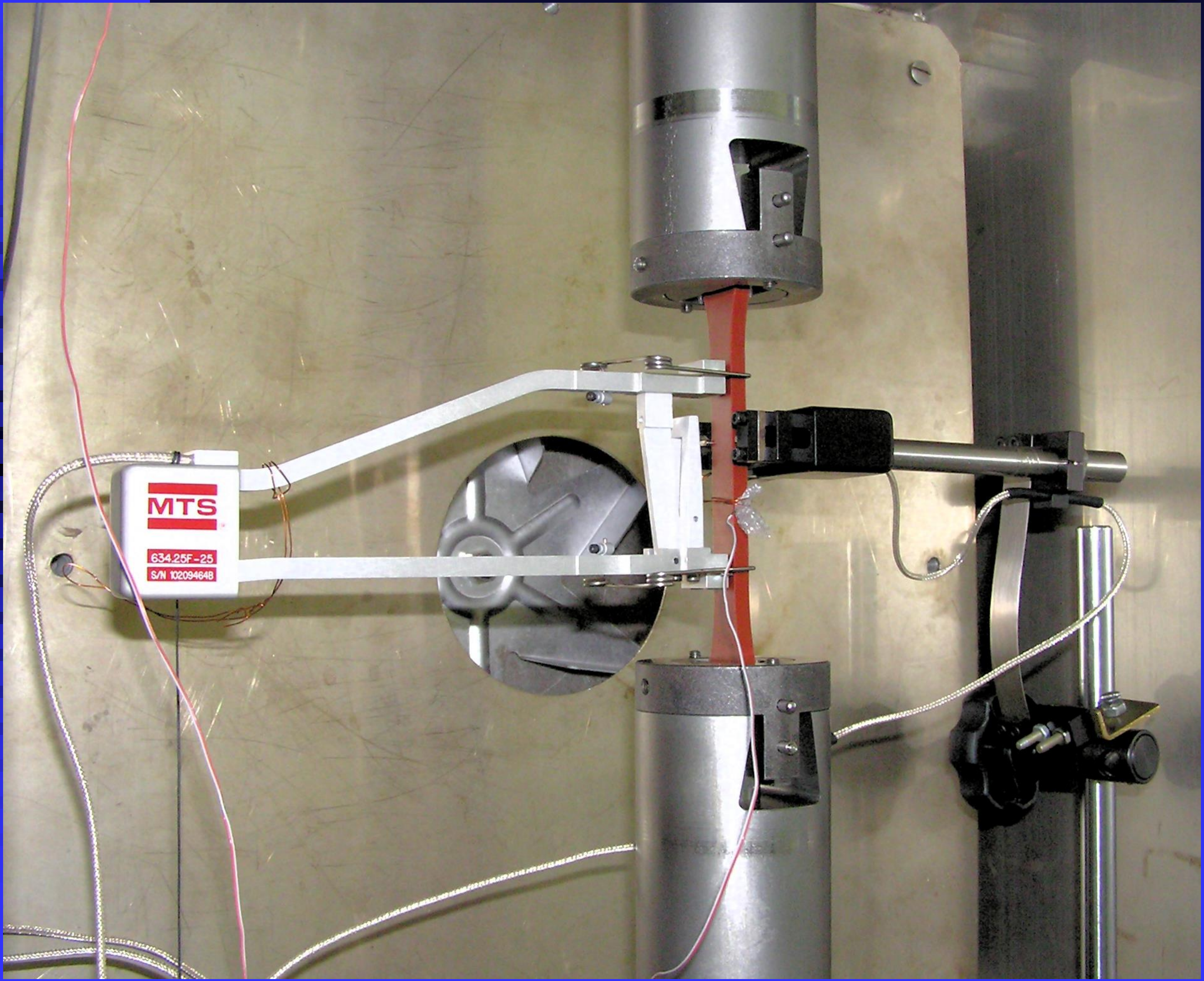
	bond rigidity	Young’s modulus
bond type	S_0	$E \approx S_0 / r_0$ [MPa]
C-C	180	1 000 000
ionic Na-Cl	9 - 21	(0.3 – 0.7) 100 000
metallic Cu-Cu	15 - 40	(0.3 – 1.5) 100 000
H - bond	2	8 000
Van der Waals	1	2 000

Valid for metals and ceramics; for selected plastics (rubber, cauochouck) by 3 order lower comparing to theory.

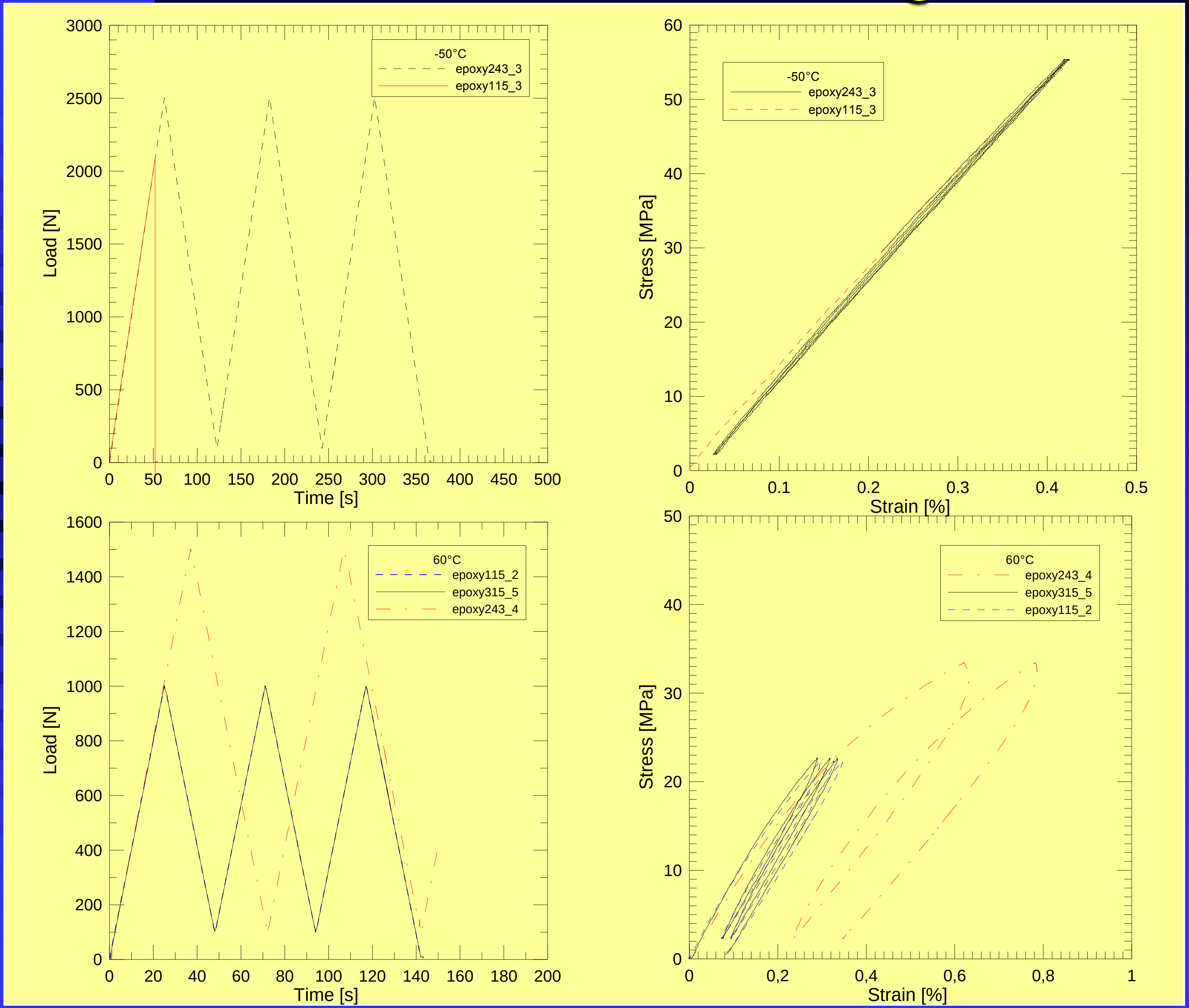
Inelasticity – internal dumping

- Deformation energy losses during vibrations
- Noise dumping

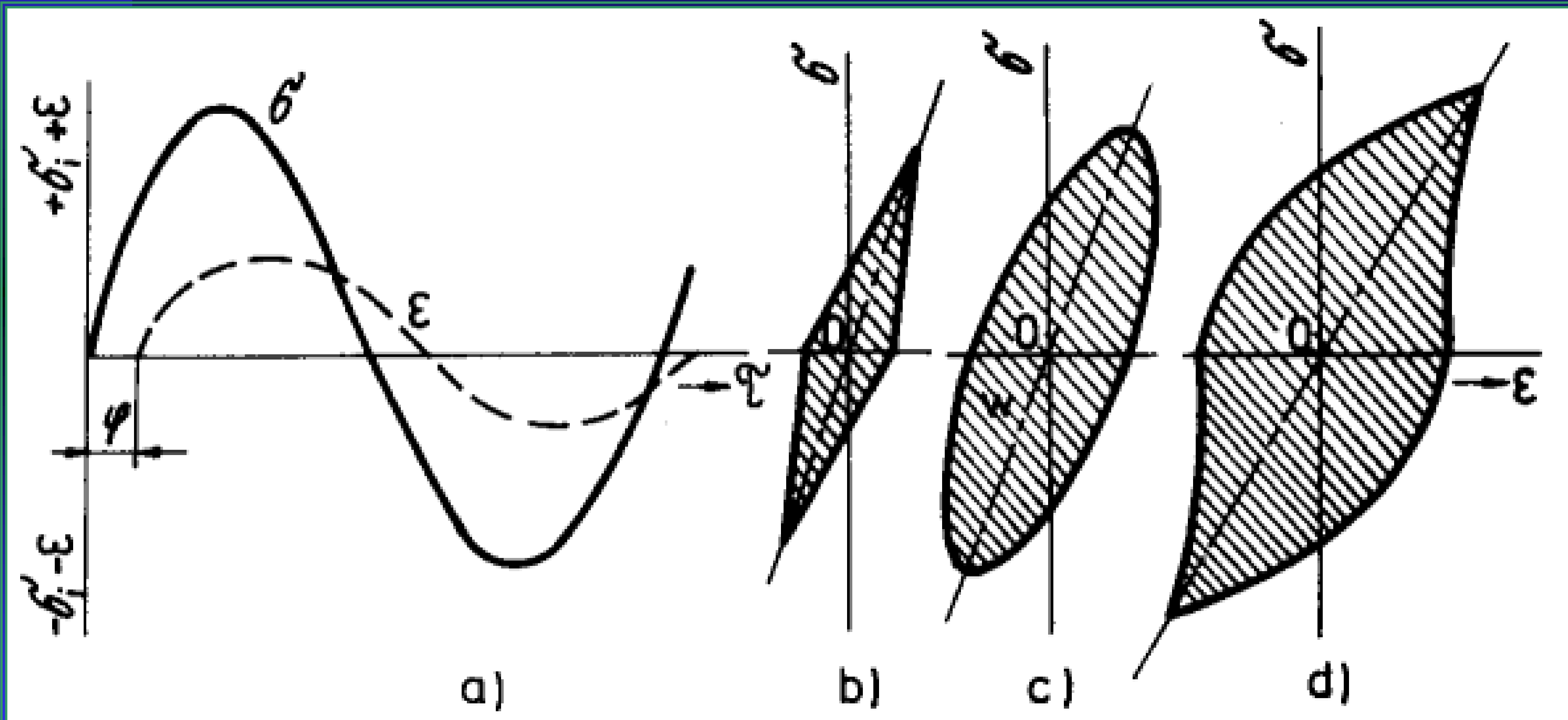
Určování modulu pružnosti



Quasistatic methods

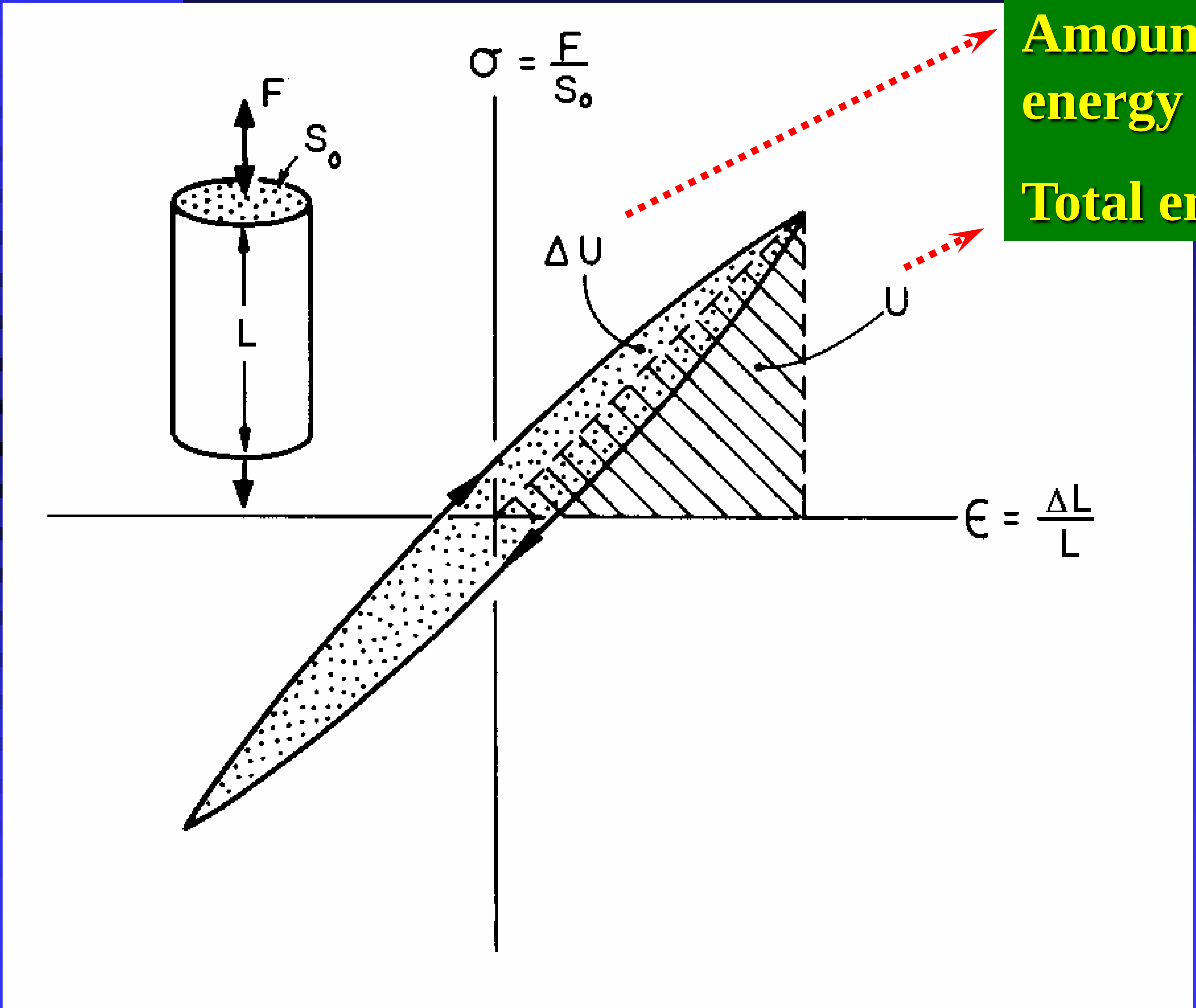


$Q^{-1} = tg\alpha$ (internal damping – coefficient of internal losses)



Ability of the materials to dissipate elastic energy during vibrations

Inelasticity / Internal damping

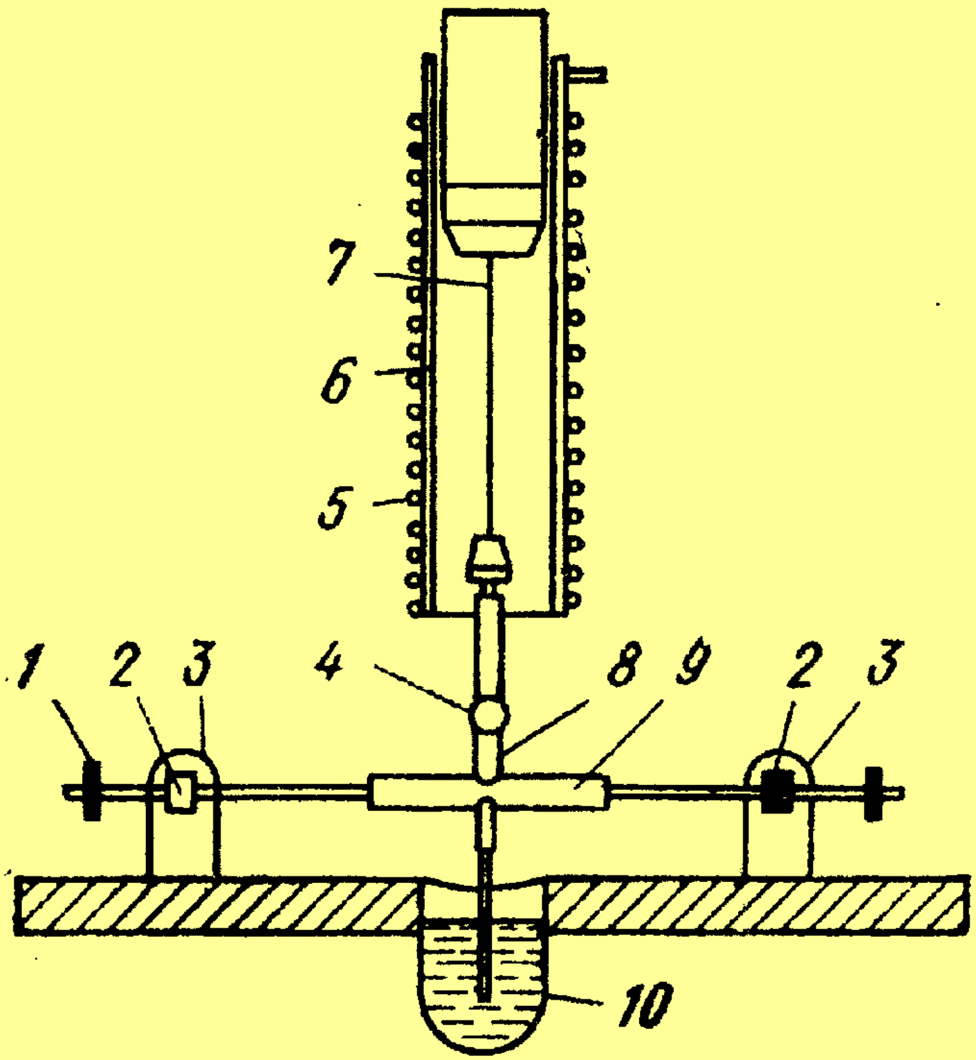


Amount of dissipated energy per one cycle

Total energy input

$$\eta = \frac{\Delta U}{2\pi U}$$

Measurements of internal damping parameters



Torzional pendulum

$$G = 128\pi . J a . l . f^2 / d^4$$

d – sample diameter
l – length of the sample
J – axial moment of inertia
f – eigenfrequency pendulum vibrations

$\Delta\omega$ – half of width on the rezonance peak in the half of its height

ω_0 – rezonance circular frequency of sample

$$\eta = \frac{\Delta U}{2\pi U} = \frac{\Delta\omega}{\sqrt{3}\omega_0}$$

Inelasticity – internal dumping

- **Ability of the material dissipate elastic energy during vibrations**
- **Internal dumping quantification**

- coefficient of internal losses

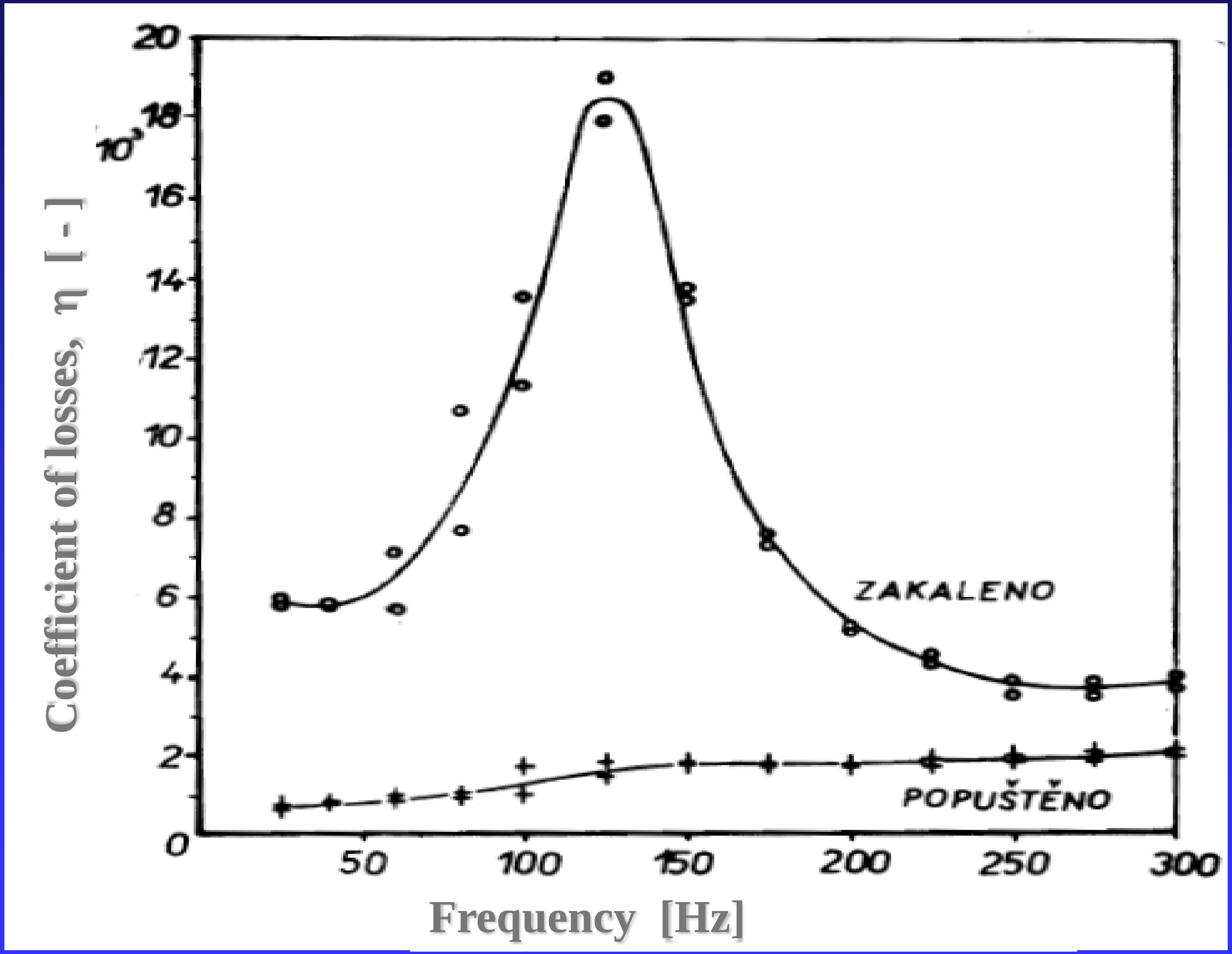
$$\eta = \frac{\Delta U}{2\pi U} \quad (Q^{-1})$$

- logarithmic decrement of dumping

$$\vartheta = \ln\left(\frac{a_n}{a_{n+1}}\right)$$

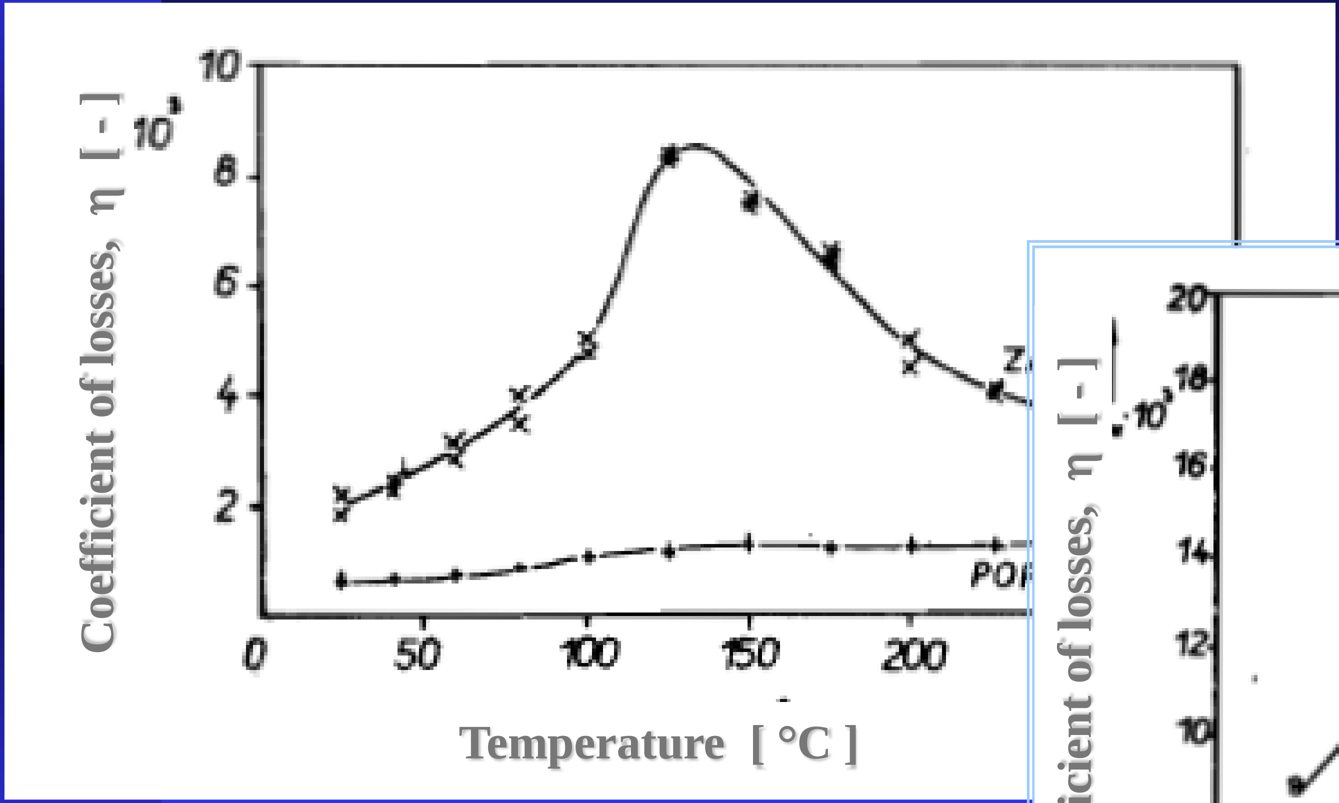
Measurements of internal damping parameters

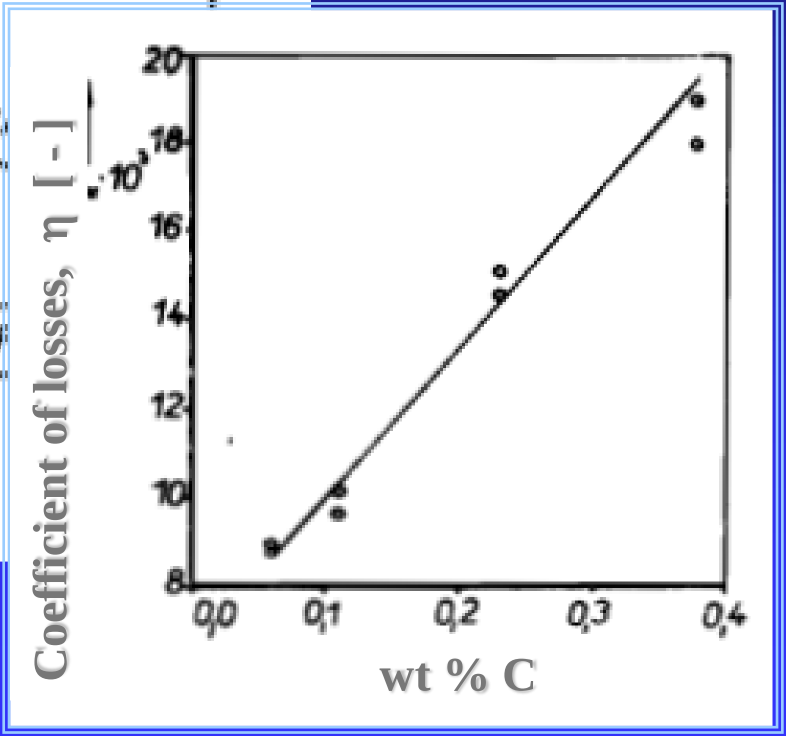
Frequency dependence of coefficient of losses



Measurements of internal damping parameters

Temperature dependence of coefficient of losses





Measurements of internal damping parameters

- mechanical spectroscopy

Inelasticity – internal dumping

- **Ability of the material dissipate elastic energy during vibrations**
- **Internal dumping quantification**

- coefficient of internal losses

- logarithmic decrement of dumping

- ***Practical meaning:***

gear box, engine block, railway wheels etc.

Inelasticity – internal dumping

Physical background

Frequency dependent internal dumping

- thermoelastic phenomenon (elongation – cooling, compression - heating)
- motion of valency electrons (at several Kelvins)
- viscous flow - grain boundaries (Ke, self difusion)
- carbon diffusion in solid solution of α-iron (Snoek)
- change of orientation of bi-vacancies etc.
- relaxation of dislocations – Bordoni peak
- dynamic properties of dislocations
- relaxation processes in plastics

Inelasticity – internal dumping

Physical background

Frequency independent internal dumping

- behaviour of dislocations segments (dislocations between anchoring points)
- magnetic-mechanic dumping (magnetic domains rotation)

Inelasticity – internal dumping

Importance

Frequency dependent internal dumping

Interesting for materials scientists rather than mechanical engineers

(phenomena conditioned by some relaxation process):

- carbon diffusion in solid solution of α-iron (Snoek)
- relaxation in dislocations (Bordoni)
- dynamic properties of dislocations and interactions of dislocations with obstacles – dislocation, atmospheres, precipitates, stacking faults)

Inelasticity – internal dumping

Importance

Frequency independent internal dumping

Key property for designers

- railway cars (wheels)
- gear box
- noise dumping etc.

Suitable materials:

- magnesium alloys – reversible twin deformation
- Mn-Cu alloy – reversible martensitic transformation
- cast iron – energy dissipation on grafite particles
- concrete – energy diss. on inhomogeneous structure

Inelasticity – internal dumping

Material selection of vibration table

(vibration table – structures testing for cars, aircrafts)

Material demands:

Inelasticity – internal dumping

Material selection of vibration table

(vibration table – structures testing for cars, aircrafts)

Material demands:

low specific weight (density) ρ

high rigidity $E > 30$ GPa

high eigenfrequency $(E/\rho)^{1/2}$

high coefficient of internal losses $\eta \approx 10^{-2}$

Material	η	$(E/\rho)^{1/2}$	ρ [Mg/m^3]	Evaluation
Mg alloys	$10^{-2} – 10^{-1}$	5.10^3	1,75	best solution
alloy Mn-Cu	10^{-1}	$3,5.10^3$	8	damping OK, but heavy
KFRP/GFRP	2.10^{-2}	6.10^3	1,8	acceptable
cast iron	2.10^{-2}	6.10^3	7,8	acceptable/heavy
concrete	2.10^{-2}	$3,5.10^3$	2,5	acceptable

Inelasticity – internal dumping