

Toughness/crack resistance

- i.** Basic notations (transition fracture behaviour of steels, temperature dependence of strength properties, fractography and failure analysis – basic approach)
- ii.** (Empirical) tests of toughness/crack resistance (Charpy, Pellini diagram, NDTT)
- iii.** Linear – elastic fracture mechanics – LEFM (Irwin, fracture toughness tests), Elastic – plastic fracture mechanics EPFM (tests, interpretation)

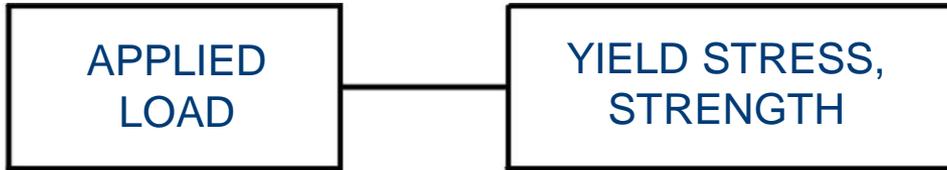
How to avoid brittle fracture of welded steel structure/component

- **crack arrest concept**

 - transition temperature

- **concept of**

 - fracture mechanics



$$\sigma_{\max} \leq R_f/k$$

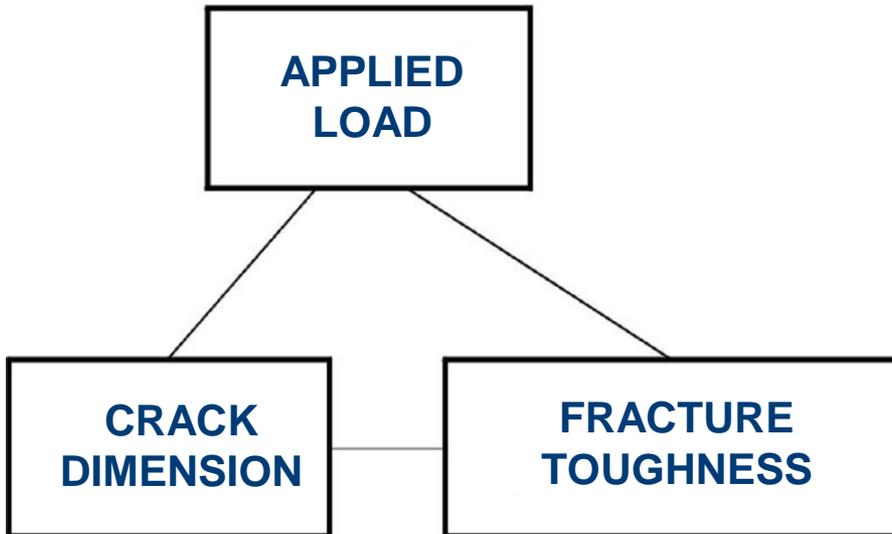
k - safety factor

R_f - strength parameter

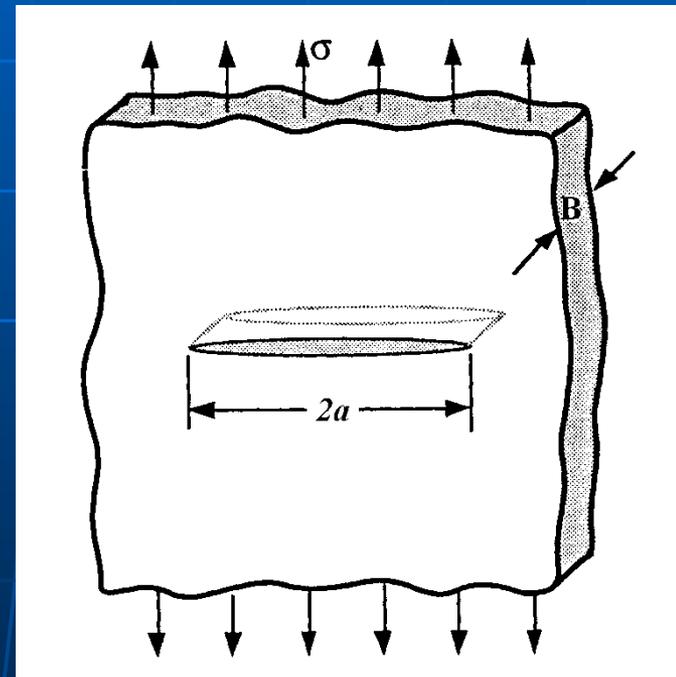
$$t_{\min} \geq t_t + \Delta t$$

t_t - transition temperature

Δt - safety factor to trans. temperature



Fracture mechanics is a scientific field dealing with limit states of bodies with cracks/defects

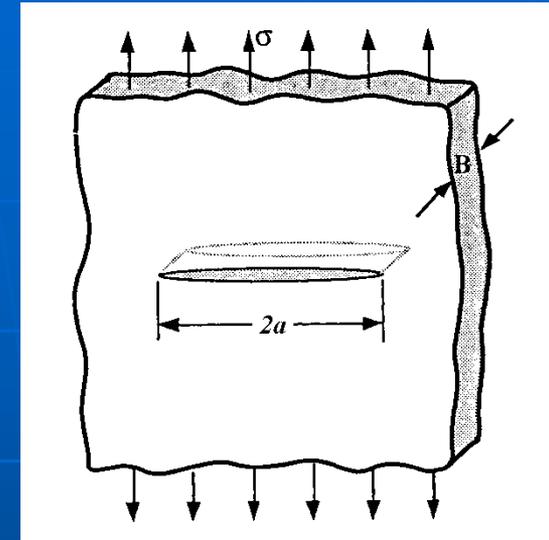


Stressed (loaded) body with a crack – **energetic analysis** – **Griffith** – parameter G

Stressed (loaded) body with a crack – **stress analysis** – **Irwin (Kinz)** – parameter K

Energetic criterion - Griffith

- ❖ Infinitely large plate loaded by constant stress, internal crack, thickness B
- ❖ Energy balance (energy conservation law)
- ❖ Work associated with new fracture surface increment (crack propagation) “covered” by elastic energy released in surrounding of the growing crack



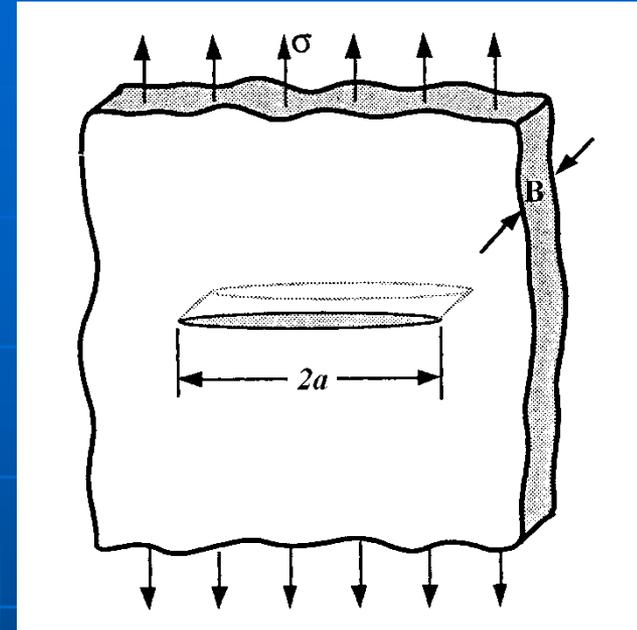
$$\frac{dW}{dA} = \frac{dW_{el}}{dA} + \frac{dW_s}{dA} \leq 0$$
$$-\frac{dW_{el}}{dA} = \frac{dW_s}{dA}$$

Energetic criterion - Griffith

Crack may propagate only if the process leads to total energy decrease and/or is the same

At accident, at sudden failures caused by brittle fracture, the work connected with fracture formation is covered by elastic energy accumulated in the structure

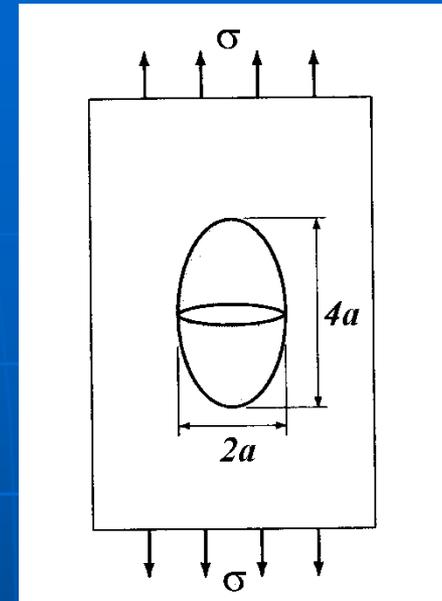
the fracture is possible only when released elastic energy at the crack tip is capable to create new surfaces



Energetic criterion - Griffith

$$\frac{dW}{dA} = \frac{dW_{el}}{dA} + \frac{dW_s}{dA} \leq 0$$

$$-\frac{dW_{el}}{dA} = \frac{dW_s}{dA}$$

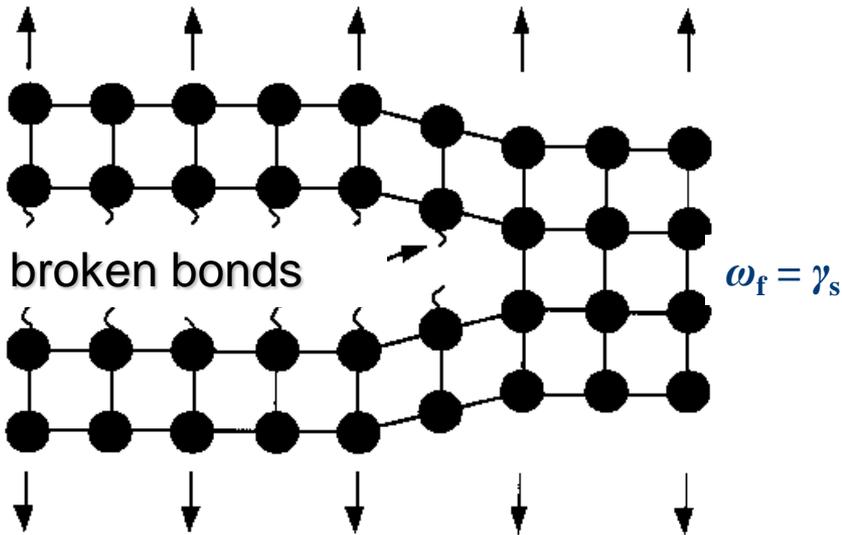


$$W_{el} = -\frac{\pi \cdot a^2 \cdot \sigma^2 \cdot B}{E}$$

$$W_s = 2 \cdot 2 \cdot a \cdot B \cdot \gamma_s$$

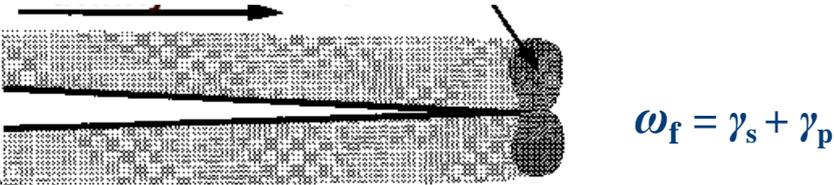
$$\sigma_f = \left(\frac{2 \cdot E \cdot \gamma_s}{\pi \cdot a} \right)^{1/2}$$

Energetic criterion – Griffith-Orowan



(a) Ideally brittle material

Crack development plastic zone



(b) Kvazibrittle, elastic plastic material

$\omega_f = \gamma_s$ $\left(\frac{\text{skutečná plocha}}{\text{průmět plochy}} \right)$



$$\sigma_f = \left(\frac{2 \cdot E \cdot \gamma_s}{\pi \cdot a} \right)^{1/2}$$

$$\sigma_f = \left(\frac{2E\omega_f}{\pi a} \right)^{1/2}$$

$$\sigma_f = \left(\frac{2E(\gamma_s + \gamma_p)}{\pi \cdot a} \right)^{1/2}$$

Energetic criterion – Griffith-Orowan

(Irwin modification of Griffiths theory)

$$G = -\frac{dW_{el}}{da}$$
$$G = \frac{\pi\sigma^2 a}{E} \left[\frac{(MPa)^2}{MPa} m = \frac{MNm}{m^2} \right]$$

MJ

***m*²**



energy release rate

MN

m



crack driving force

Energetic criterion – Griffith-Orowan

$$\frac{dW_s}{da} = 2\omega_f = 2(\gamma_s + \gamma_p)$$

$2\omega_f$ \rightarrow G_c – fracture toughness of the material
 $2\omega_f$ \rightarrow R (R - curve) - materials crack resistance

Energetic criterion – Griffith-Orowan

CRACK DRIVING FORCE G

Energy release rate (rate of potential energy change in dependence on fracture surface increment)

$$G = -\frac{dW_{el}}{dA} \quad G = \frac{\pi\sigma^2 a}{E} \quad \left[\frac{(MPa)^2}{MPa} m = \frac{MJ}{m^2} = \frac{MN}{m} \right]$$

MATERIALS CRACK RESISTANCE R

Rate of surface energy increase with increase of fracture surfaces, the critical value – condition for crack initiation

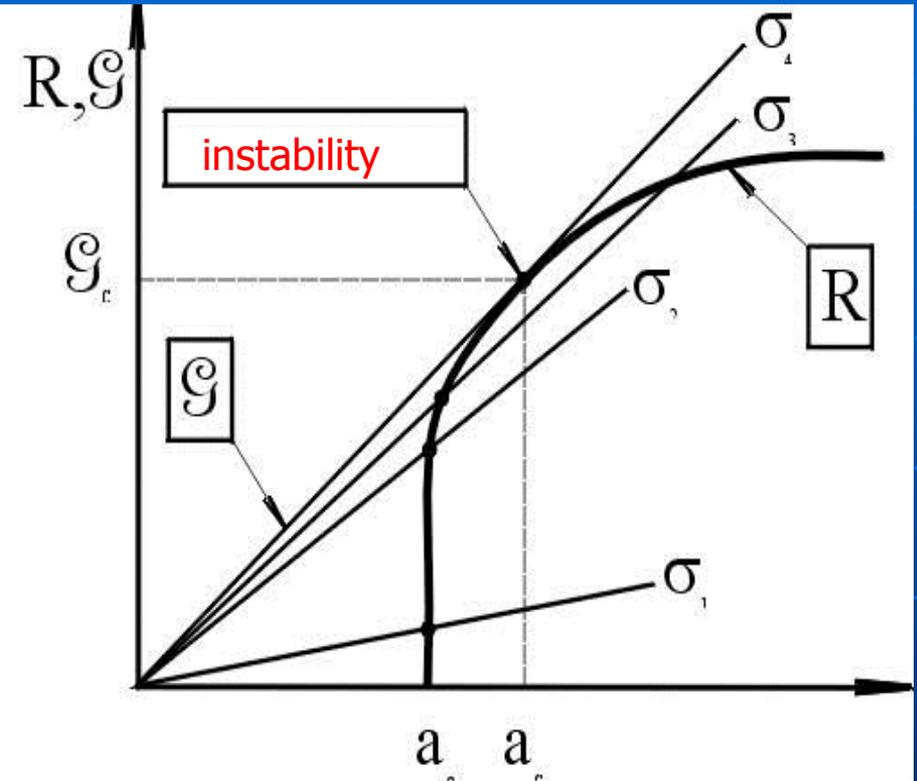
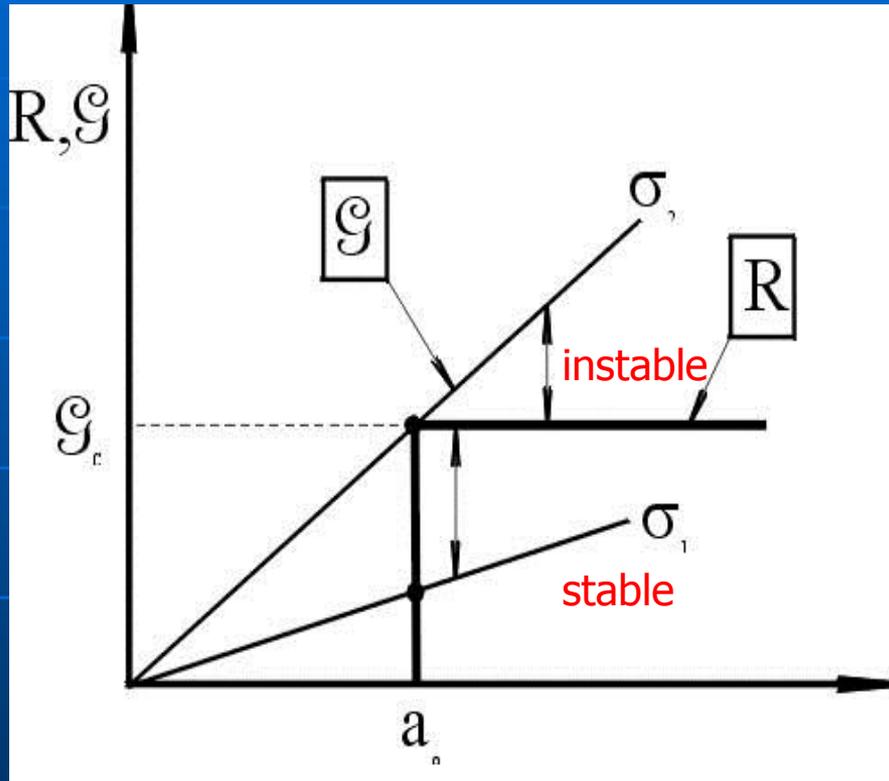
$$R = \frac{dW_s}{dA} \quad R = 2(\gamma_s + \gamma_p) = G_C$$

toughness G_C

Energetic criterion – Griffith-Orowan

Crack resistance curve – inherent property of the material

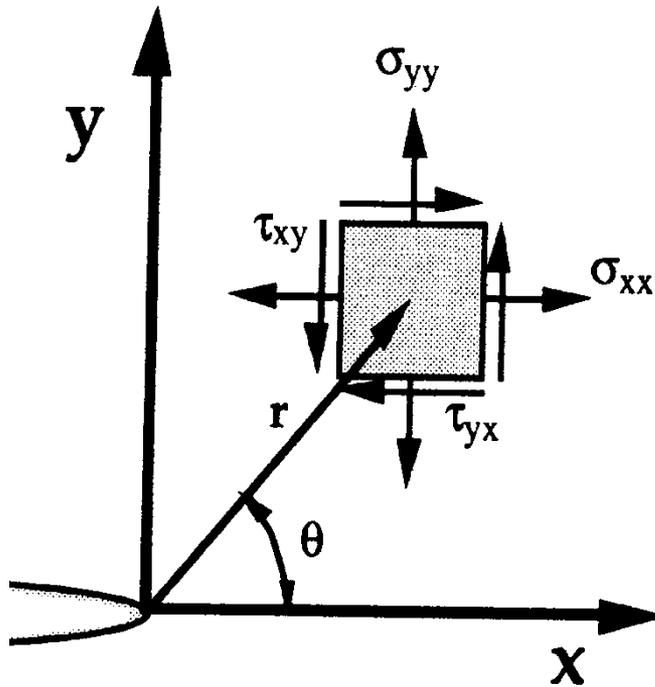
G_C – material property (fracture toughness)



stable crack propagation – crack does not propagate, as far as applied load increases

unstable crack propagation – crack propagates spontaneously, without necessity of further load increasing

Stress criteria - Irwin



r, θ - polar coordinates

σ_{ij} - components of stress tensor

k - constant

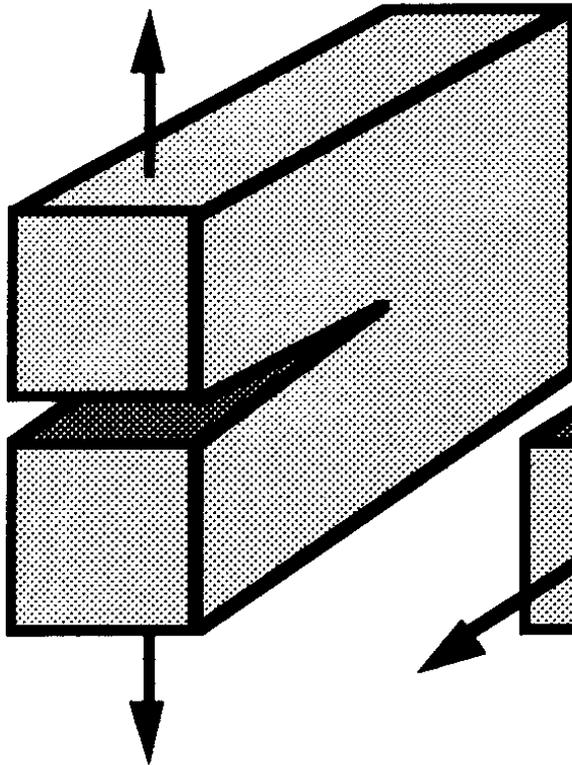
$f_{ij}(\theta), g_{ij}(\theta)$

- dimensionless quantities
(function of angle θ)

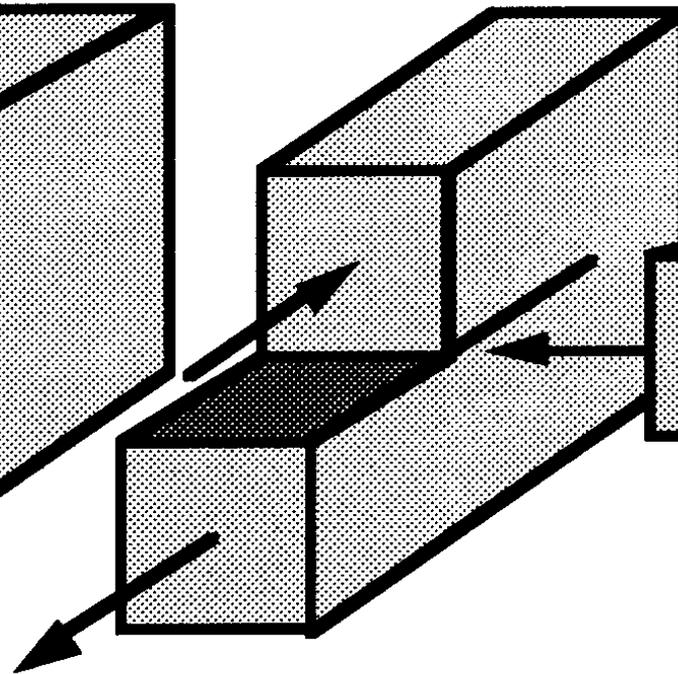
$$\sigma_{ij} = \left(\frac{k}{\sqrt{r}} \right) \cdot f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m \cdot r^{\frac{m}{2}} g_{ij}^{(m)}(\theta)$$

loading modes

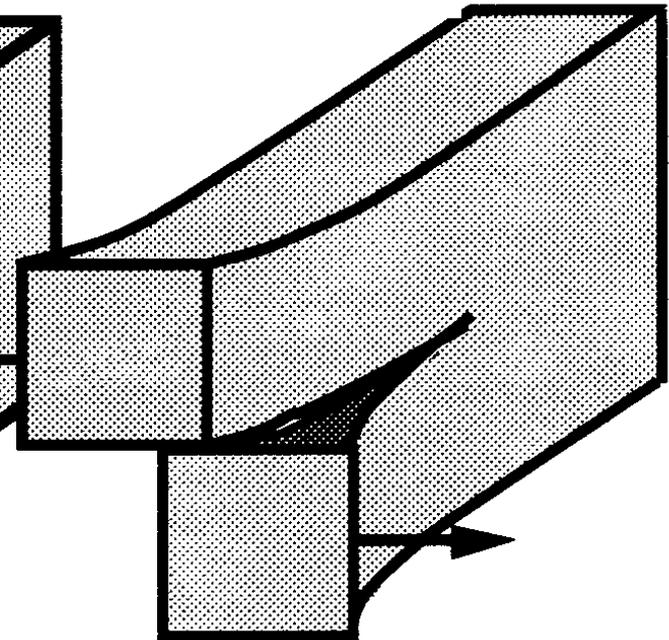
I. opening



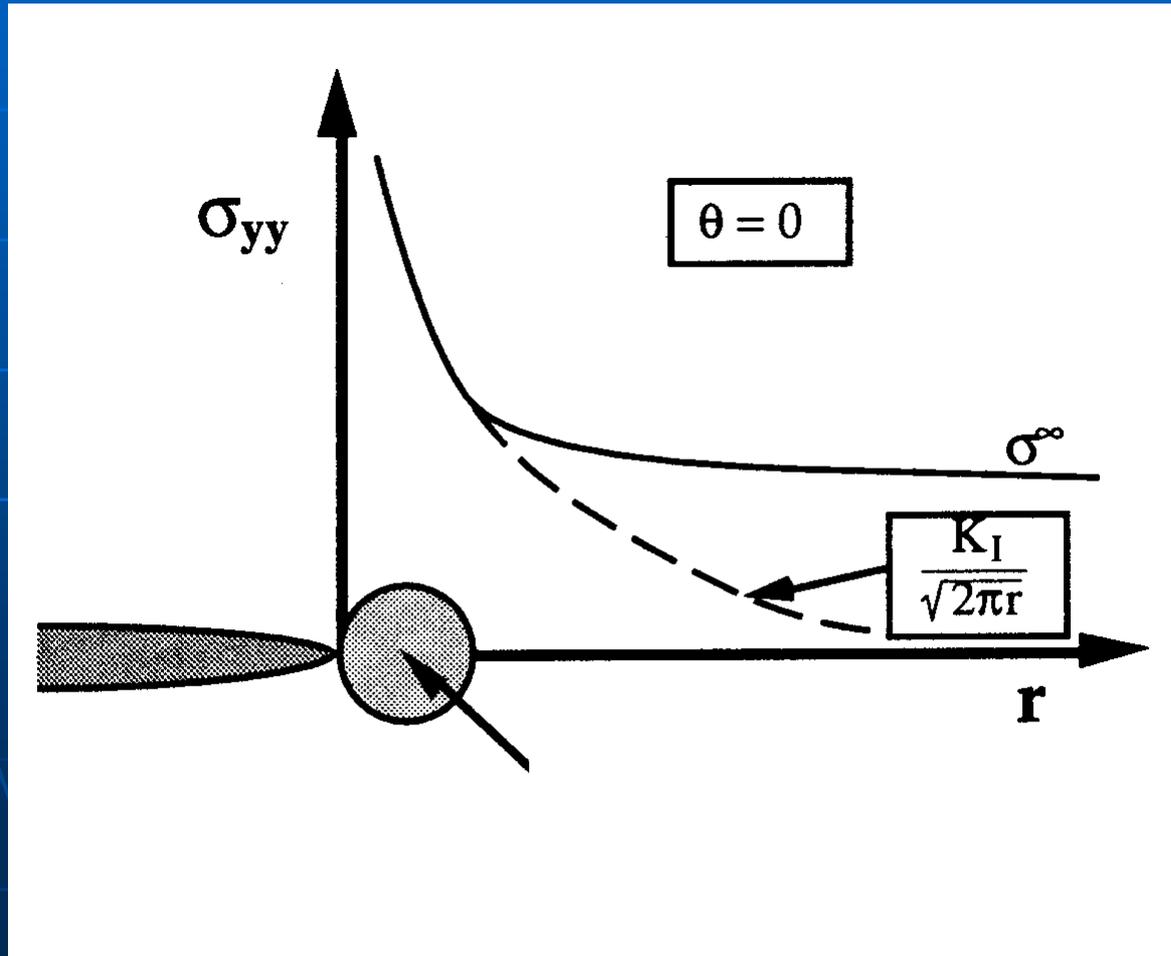
II. shear



III. torsion



Stress criteria - Irwin



stress intensity factor

$$\sigma_{ij} = \left(\frac{k}{\sqrt{r}} \right) \cdot f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m \cdot r^{\frac{m}{2}} g_{ij}^{(m)}(\theta)$$

$$\sigma_{ij} = \left(\frac{k}{\sqrt{r}} \right) \cdot f_{ij}(\theta)$$

$$K_I = k \cdot \sqrt{2\pi}$$

Stress criteria - Irwin

Napětíová pole před čelem trhliny pro Mód I a Mód II v lineárním elastickém a isotropním materiálu

	Mód I	Mód II
σ_{xx}	$\frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right]$	$-\frac{K_{II}}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right) \left[2 + \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \right]$
σ_{yy}	$\frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right]$	$\frac{K_{II}}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right)$
τ_{xy}	$\frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right)$	$\frac{K_{II}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right]$
σ_{zz}	0(rovinná napjatost) $\mu(\sigma_{xx} + \sigma_{yy})$ (rovinná deformace)	0(rovinná napjatost) $\mu(\sigma_{xx} + \sigma_{yy})$ (rovinná deformace)
τ_{xz}	0	0
τ_{yz}		

Stress criteria - Irwin

Posunutí před čelem trhliny pro Mód I a Mód II v lineárním elastickém a isotropním materiálu

	Mód I	Mód II
u_x	$\frac{K_I}{2G} \sqrt{\frac{r}{2\pi}} \cos\left(\frac{\theta}{2}\right) \left[\kappa - 1 + 2 \sin^2\left(\frac{\theta}{2}\right) \right]$	$\frac{K_{II}}{2G} \sqrt{\frac{r}{2\pi}} \sin\left(\frac{\theta}{2}\right) \left[\kappa + 1 + 2 \cos^2\left(\frac{\theta}{2}\right) \right]$
u_y	$\frac{K_I}{2G} \sqrt{\frac{r}{2\pi}} \sin\left(\frac{\theta}{2}\right) \left[\kappa + 1 - 2 \cos^2\left(\frac{\theta}{2}\right) \right]$	$-\frac{K_{II}}{2G} \sqrt{\frac{r}{2\pi}} \cos\left(\frac{\theta}{2}\right) \left[\kappa - 1 - 2 \sin^2\left(\frac{\theta}{2}\right) \right]$

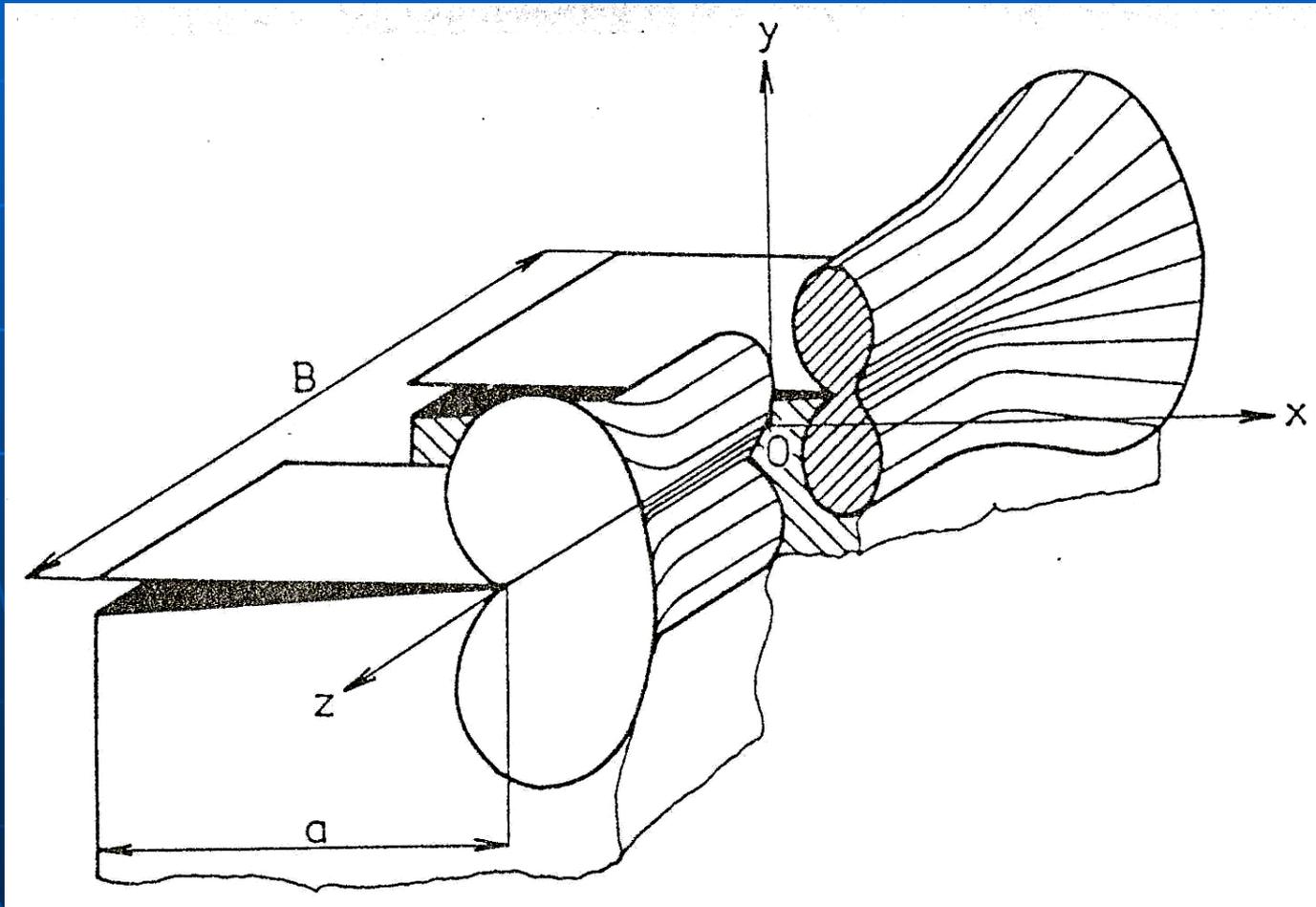
G je modul ve smyku, $\kappa = 3-4\mu$ (rovinná deformace), $\kappa = (3-\mu)/(1+\mu)$

Tab. 3 Složky nenulových napětí a posunutí v Módu III v lineárním elastickém a isotropním materiálu

$\tau_{xz} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right)$	$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)$	$u_z = -\frac{K_{III}}{G} \sqrt{\frac{r}{2\pi}} \sin\left(\frac{\theta}{2}\right)$
--	---	--

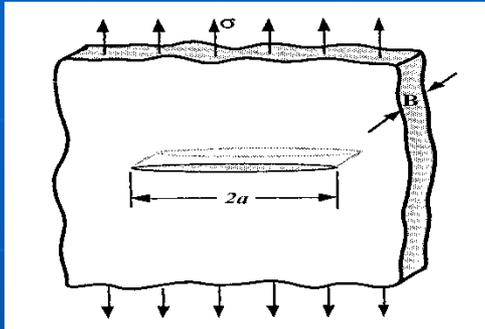
Stress distribution - biaxial vs. triaxial

Strain distribution – triaxial vs. biaxial

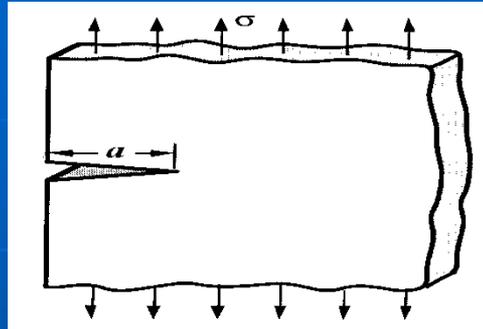


Stress criteria - Irwin

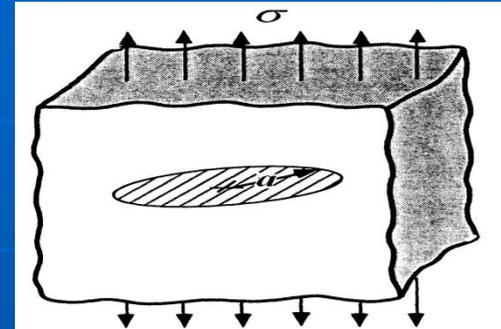
- Basic solutions



$$K_I \approx \sigma$$



$$K_I = \sigma \sqrt{\pi \cdot a}$$



$$K_I = \frac{2}{\pi} \sigma \sqrt{\pi \cdot a}$$

- General solutions (tabeled)

$$K_I = \frac{F}{B \cdot \sqrt{W}} \cdot f\left(\frac{a}{W}\right)$$

B – thickness

W – width / dimension in crack propagation direction

a – crack length

F – fracture force

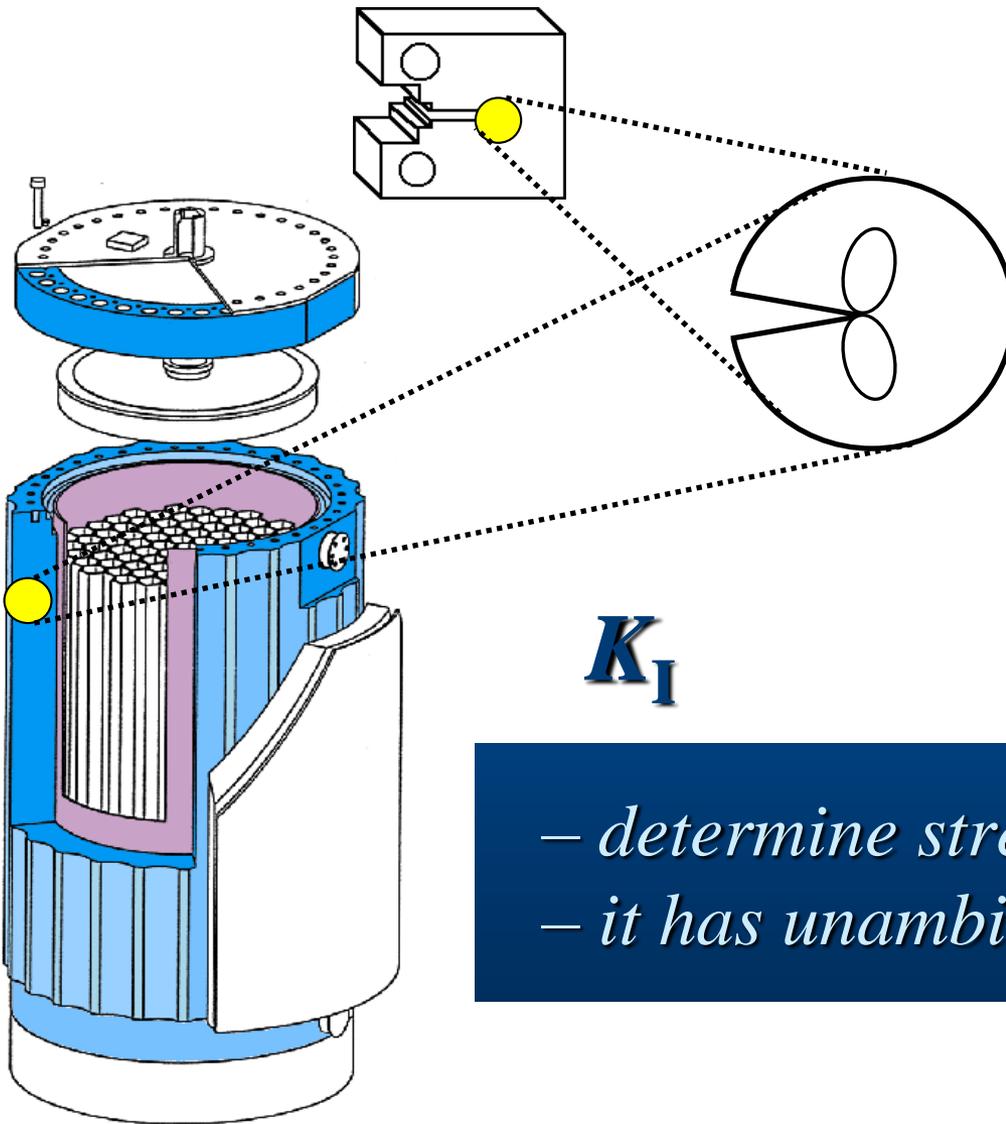
units K_I

$$K_I = \sigma \sqrt{\pi \cdot a} \quad \text{MPa}\sqrt{\text{m}} = \text{MPam}^{1/2} = \text{MPam}^{0,5}$$

relationship between G_C a K_{IC}

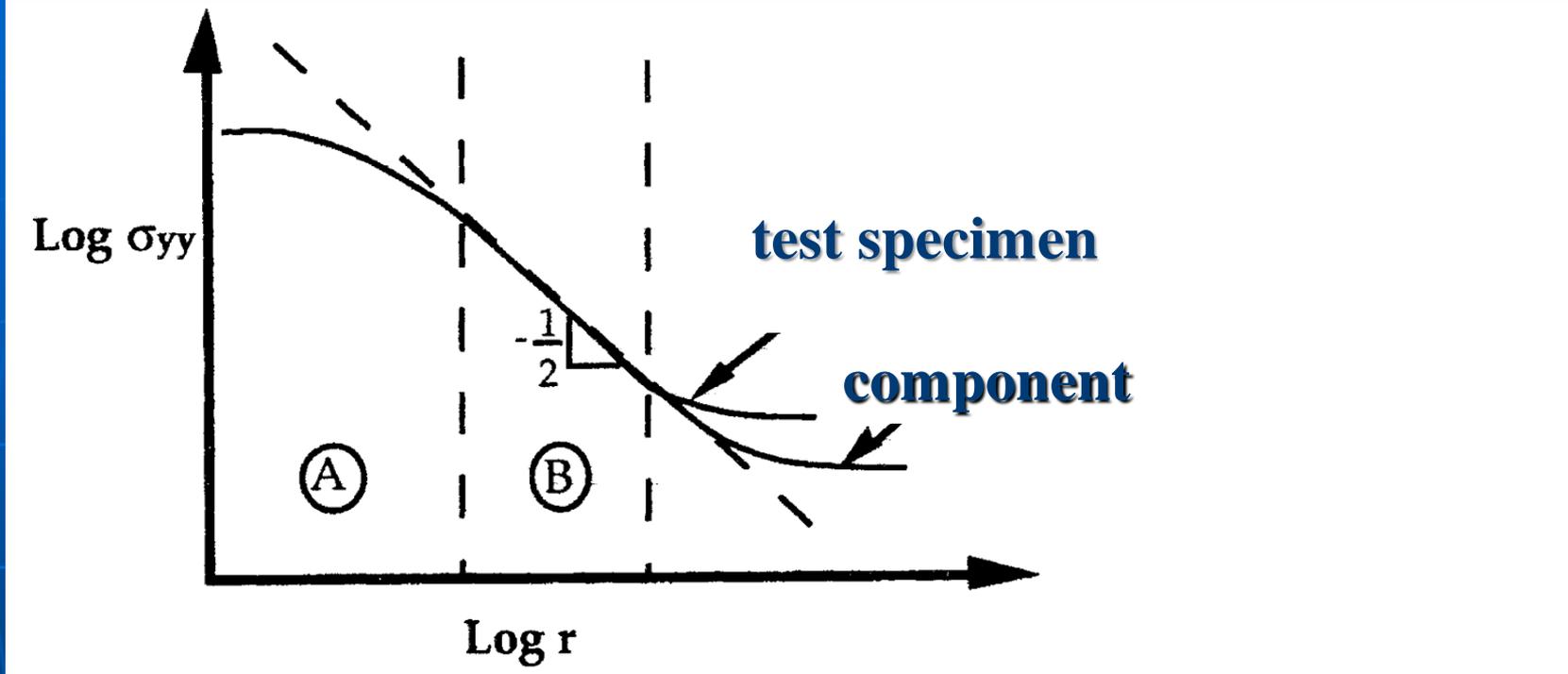
$$G = \frac{\pi \sigma^2 a}{E} \quad \longrightarrow \quad G = \frac{K_I^2}{E} \quad \longleftarrow \quad K_I = \sigma \sqrt{\pi \cdot a}$$
$$G = \frac{K_I^2}{E} (1 - \nu^2)$$

validity conditions



K_I

- *determine stress state at the crack tip*
- *it has unambiguous relation to G_C*



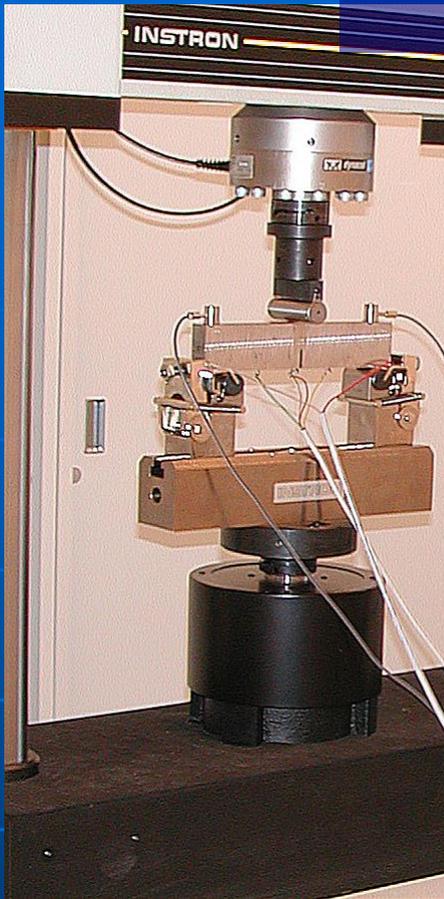
Fracture occurs when the stress intensity factor at the crack tip reaches the critical value $K_I \equiv K_C$ providing the region (process zone) where singularity dominates is the same

Linear elastic fracture mechanics – if the fracture occurs at small plastic zone (small scale yielding conditions, 2% velikosti tloušťky).

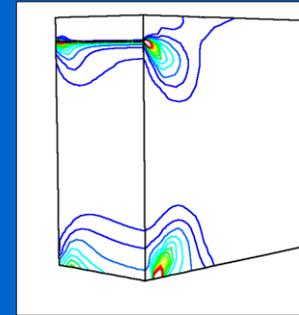
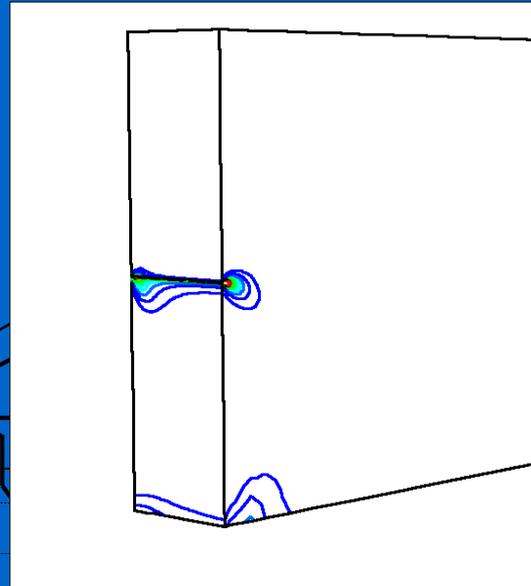
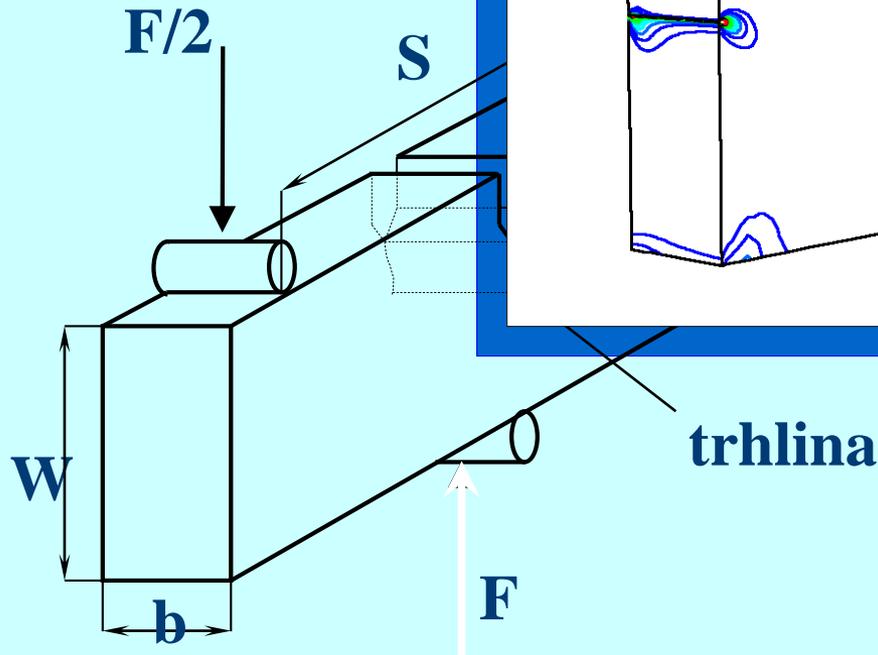
$$F_{fr} \leq (0,6 \div 0,8) F_{GY}$$

(ceramics, some plastics, aluminium alloys, high strength steels, for common steels only for very large thickness and/or dynamic loading).

determination of K_{Ic}



determination of K_{Ic}



PEEQ (Ave. Crit.: 75%)	
+	+1.000e-02
+	+9.250e-03
+	+8.500e-03
+	+7.750e-03
+	+7.000e-03
+	+6.250e-03
+	+5.500e-03
+	+4.750e-03
+	+4.000e-03
+	+3.250e-03
+	+2.500e-03
+	+1.750e-03
+	+1.000e-03

Stejný neporušený průřez

$W=50$ mm

$a/W=0,5$

$b=25$ mm

$S=200$ mm

$W=26$ mm

$a/W=0,1$

$b=25$ mm

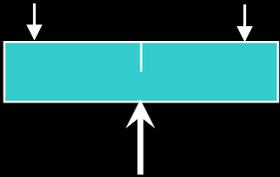
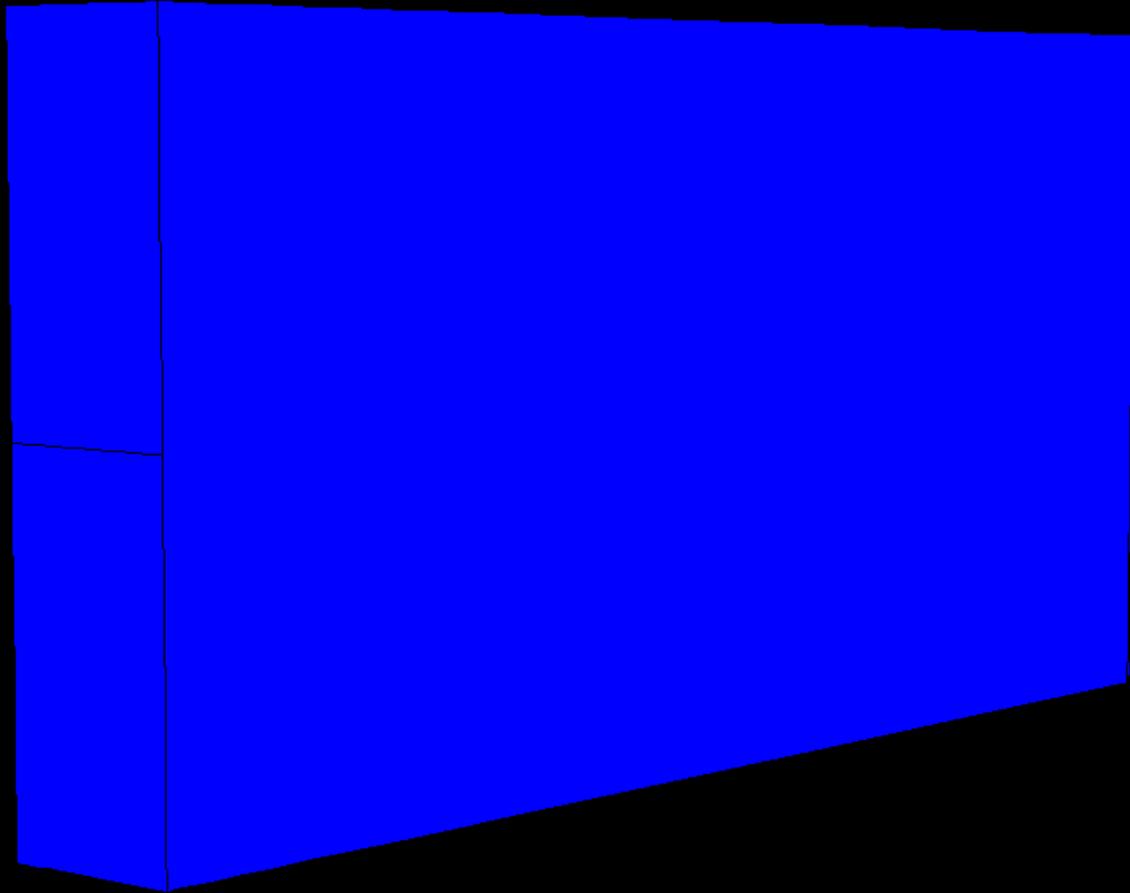
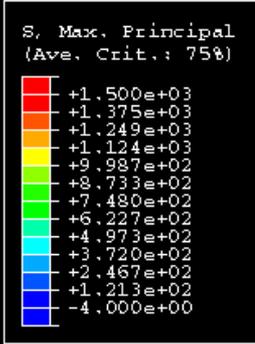
$S=104$ mm

$a/W \approx 0,5$

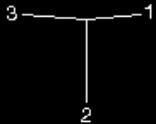
maximum principal stress

Viewport: 1 ODB: /usr/people/chlup/scratch/w05.odb

Step: 0 Frame: 0



ODB: w05.odb ABAQUS/Standard 6.1-1 Sun Mar 18 00:06:18 CET 2001



Step: load-1 Increment 0; Step Time = 0.0000E+00
Primary Var: S, Max. Principal

$a/W \approx 0,5$

maximum principal stress / detail

Viewport: 1 ODB: /usr/people/chlup/scratch/w05.odb

Step: 0 Frame: 0

S, Max. Principal
(Ave. Crit.: 75%)



ODB: w05.odb ABAQUS/Standard 6.1-1 Sun Mar 18 00:06:18 CET 2001

Step: load-1 Increment 0: Step Time = 0.0000E+00
Primary Var: S, Max. Principal

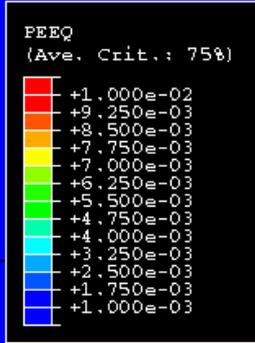


$a/W \approx 0,5$

plastic deformation - detail

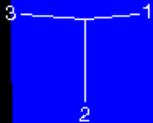
Viewport: 1 ODB: /usr/people/ch1up/scratch/w05.odb

Step: 0 Frame: 0



ODB: w05.odb ABAQUS/Standard 6.1-1 Sun Mar 18 00:06:18 CET 2001

Step: load-1 Increment 0: Step Time = 0.0000E+00
Primary Var: PEEQ

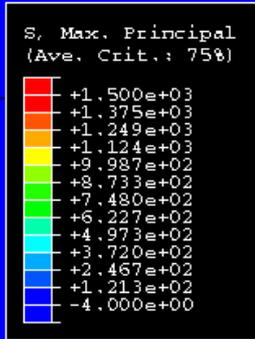


$a/W \approx 0,1$

maximum principal stress - detail

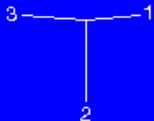
Viewport: 1 ODB: /usr/people/chlup/scratch/w01.odb

Step: 0 Frame: 0



ODB: w01.odb ABAQUS/Standard 6.1-1 Wed Apr 11 19:06:11 CEST 2001

Step: load-1 Increment 0: Step Time = 0.0000E+00
Primary Var: S, Max. Principal

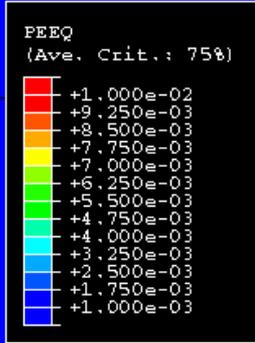


$a/W \approx 0,1$

plastic deformation - detail

Viewport: 1 ODB: /usr/people/chlup/scratch/w01.odb

Step: 0 Frame: 0



ODB: w01.odb ABAQUS/Standard 6.1-1 Wed Apr 11 19:06:11 CEST 2001

Step: load-1 Increment 0: Step Time = 0.0000E+00
Primary Var: PEEQ

