

# Fatigue of metallic materials

(Libor Pantělejev)

- 1) History of fatigue
- 2) Loading modes
- 3) Stages of fatigue life
- 4) Initiation of fatigue cracks
- 5) Life time curves
- 6) Influence of mean stress
- 7) Lifetime prediction

Acc.V Spot Magn Det WD Exp |-----| 1 mm  
20.0 kV 4.7 25x SE 33.1 84195 IGW C040

# 1) History of fatigue

1837 – Albert – first results of fatigue test (Clausthal)

1842 – Rankine, York – fatigue strength of railway's axle (London)

1853 – Morin – “Resistance des Matériaux” – „safe life approach” – inspection of axles of horse-drawn mail coaches (Paris)

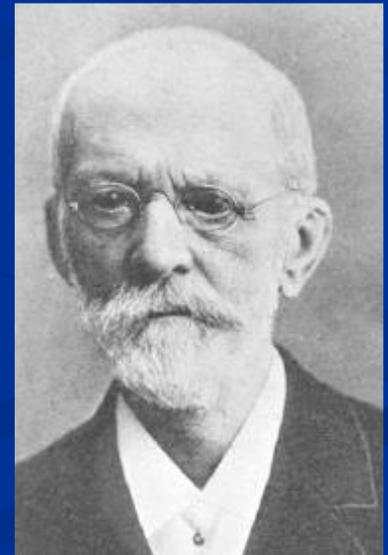
1854 – Braithwaite – term “fatigue” was mentioned first time (description of a number service fatigue failures – water pumps, propeller shafts, levers, cranes etc.)

1858 – Wöhler – wide spread measurement of loading during service of four-wheeled and six-wheeled freight and passenger cars (results published in 1860)

1870 – “Wöhler’s law” – “Material can be induced to fail by many repetitions of stresses, all of which are lower than the static strength. The stress amplitudes are decisive for the destruction of the cohesion of the material. The maximum stress is of influence only in so far as the higher it is, the lower are the stress amplitudes which lead to failure”.

Wöhler therefore stated the stress amplitudes to be the most important parameter for fatigue life, but a tensile mean stress also to have a detrimental influence.

August Wöhler  
(1819 – 1914)



Followers:

Smith, Haigh, Palmgren, Miner, Paris, Klesnil and others

## 2) Fatigue test at different loading modes:

Load (stress) control regime:  $\sigma_a = \text{const.}$

high-cycle fatigue (Wöhler, Basquin)

Testing machine:

resonant testing machine

servo-hydraulic testing machine

ultrasonic testing machine (ultra-high cycle fatigue)



Strain control regime:  $\epsilon_a = \text{const.}$

Low-cycle fatigue (Manson – Coffin)

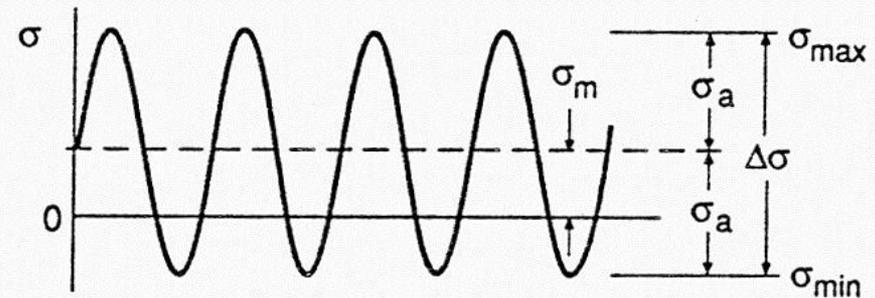
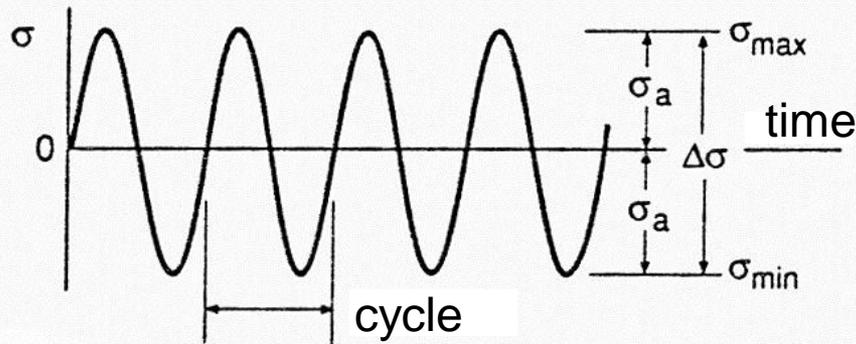
Testing machine:

servo-hydraulic testing machine



# Basic characteristics of loading cycle

- resonant testing machines are working with sinusoidal loading cycle
- servo-hydraulic testing machines allow selection of type of the loading cycle e.g. triangular, trapezoidal, saw-like etc.



$\Delta\sigma$  – stress range ( $\Delta\sigma = 2\sigma_a$ )

$\sigma_a$  – stress amplitude

$\sigma_{\max}$  – maximum stress

$\sigma_{\min}$  – minimum stress

$\sigma_m$  – mean stress

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

## Strain controlled regime:

$\Delta\varepsilon$  – strain range ( $\Delta\varepsilon = 2\varepsilon_a$ )

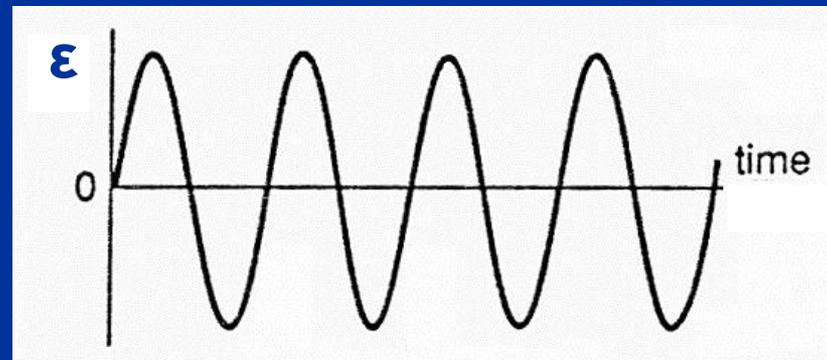
$\varepsilon_a$  – strain amplitude

$\varepsilon_{\max}$  – maximal strain

$\varepsilon_{\min}$  – minimal strain

$\varepsilon_m$  – mean strain

Testing is conducted usually in symmetrical loading cycle



# Parameters of loading cycle asymmetry

Stress ratio

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

$$P = \frac{\sigma_{\max}}{\sigma_a}$$

$$A = \frac{\sigma_a}{\sigma_m}$$

Amplitude ratio

$$P = \frac{2}{1-R}$$

$$A = \frac{1-R}{1+R}$$

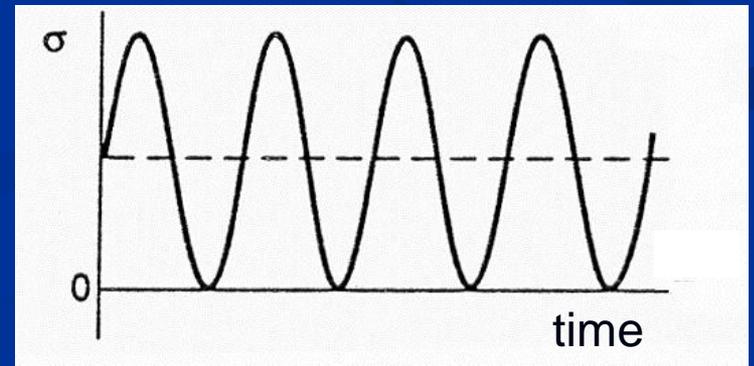
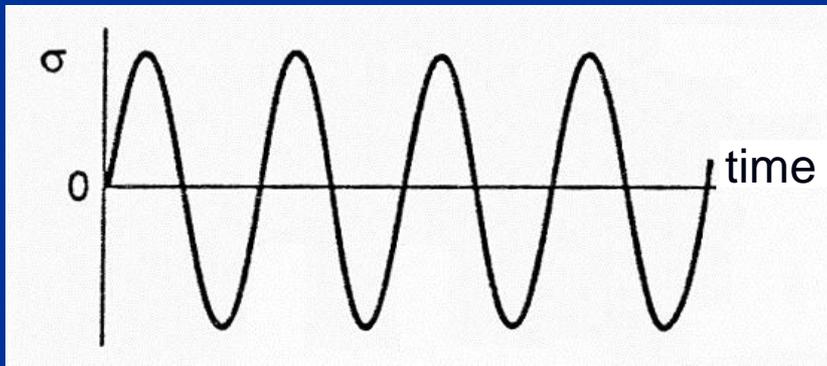
$$A = \frac{1}{P-1}$$

Symmetrical cycle:

$R = -1$ ,  $P = 1$ ,  
can not be described via parameter  $A$  –  
problem of singularity (division by zero)

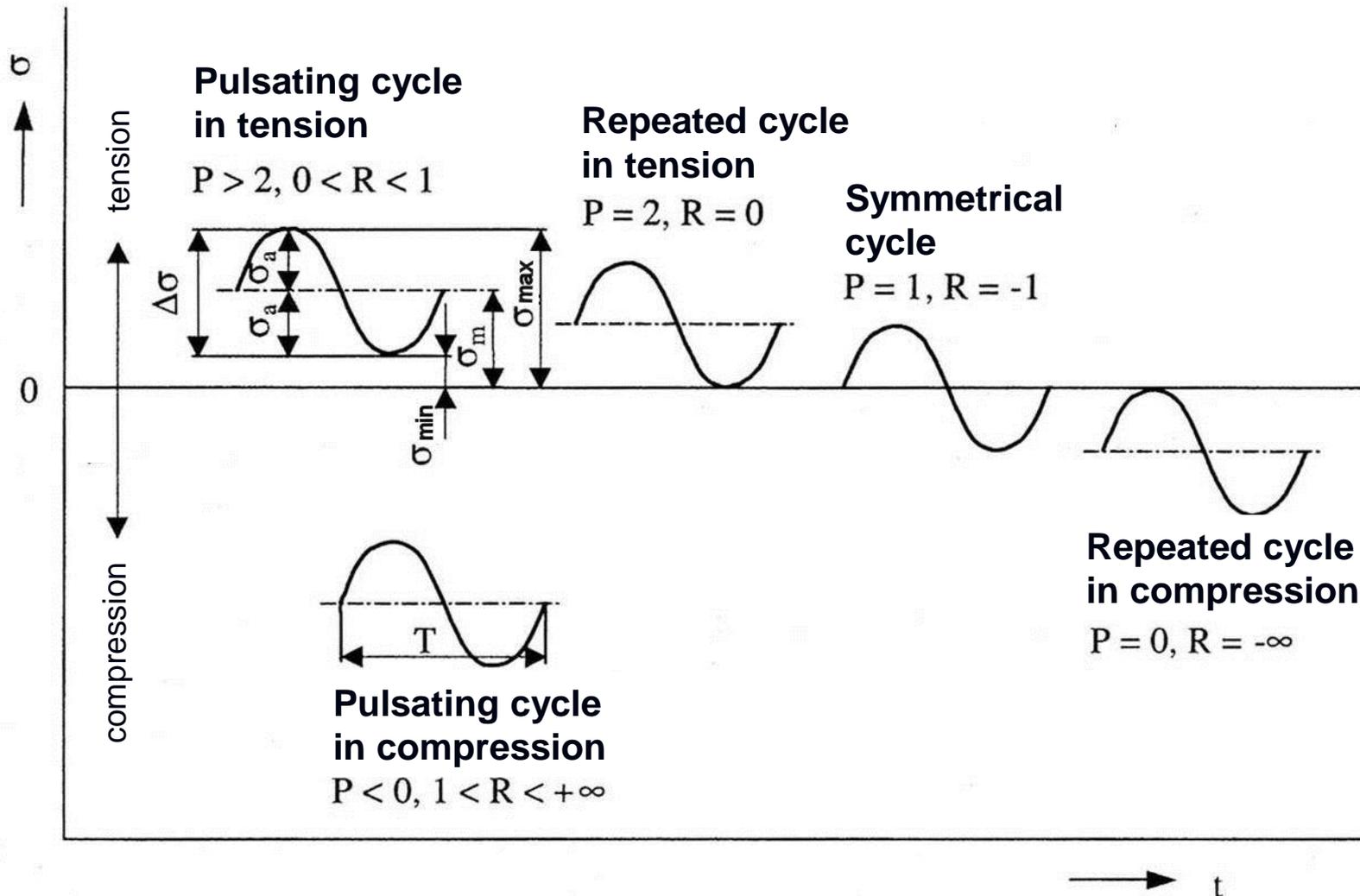
Repeated cycle:

$R = 0$ ,  $P = 2$ ,  $A = 1$



$$P = \frac{\sigma_{\max}}{\sigma_a}$$

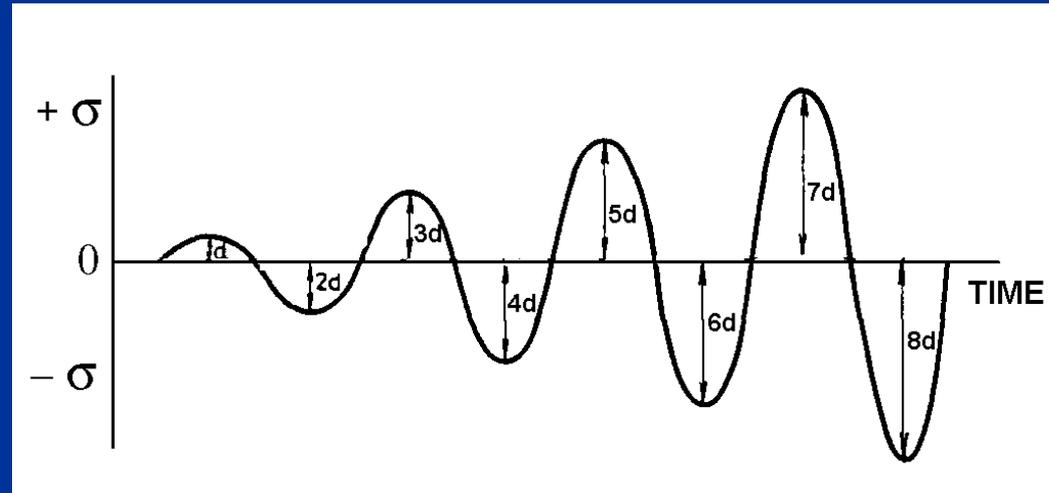
$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$



# how to reach full load

## Resonant testing machine:

- loading frequency is given by stiffness of the system (testing machine – tested specimen) – there is possibility partly to change the loading frequency by operator (add or remove of additional weight)
- full load is not reached immediately – loading ramp is inherent property of apparatus

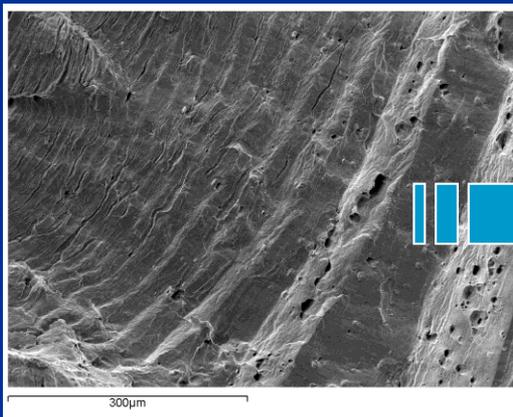
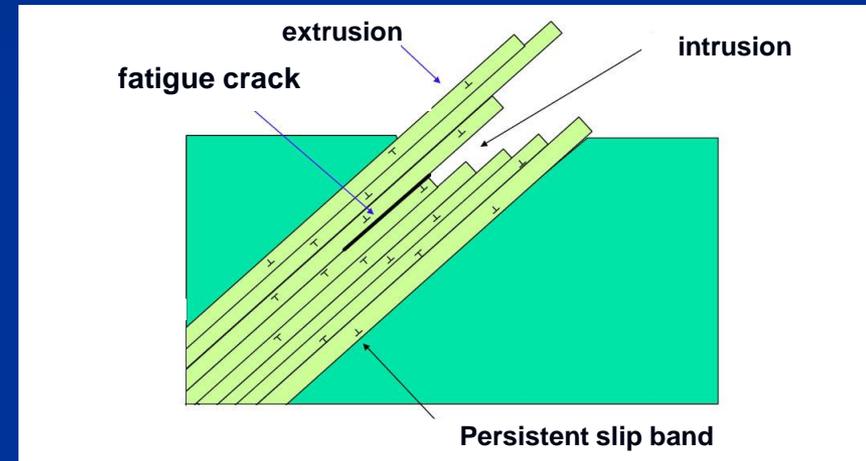
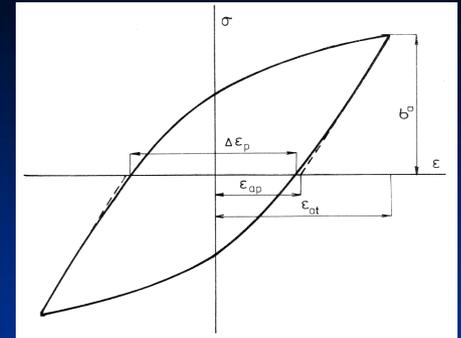


## Servo-hydraulic testing machine:

- the loading ramp/full load can be reached immediately
- loading ramp: either number of cycles to reach full load or selection of the loading frequency (in the range given by testing machine).

### 3) Stages of fatigue life

- Changes of mechanical properties  
cyclic softening/ hardening
- Crack initiation
  - surface layers
  - inhomogeneity
- Crack propagation



Final fracture

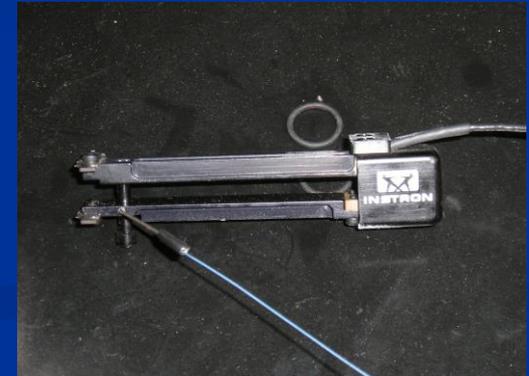
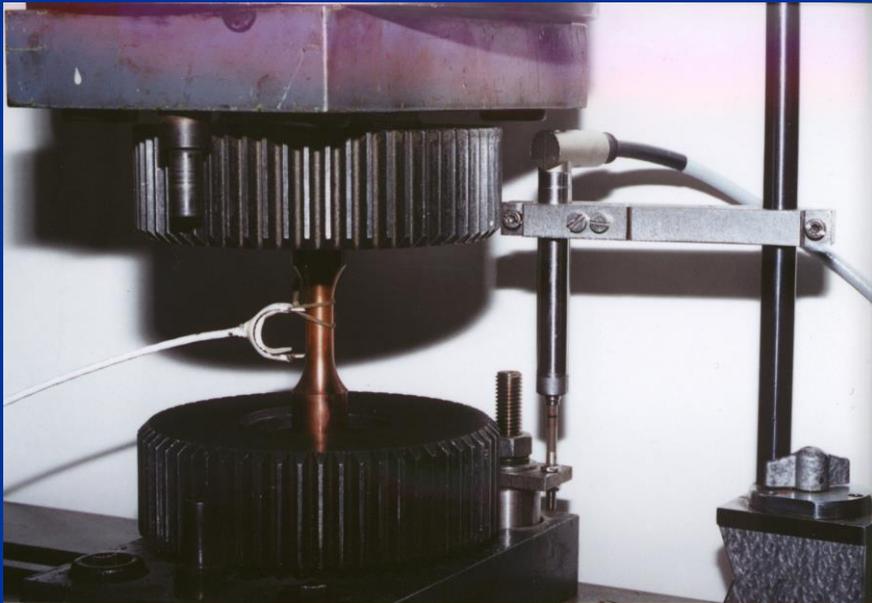


# Changes of mechanical properties

## Measurement of cyclic stress-strain response (hysteresis loop)

„clip-on“ extensometer:

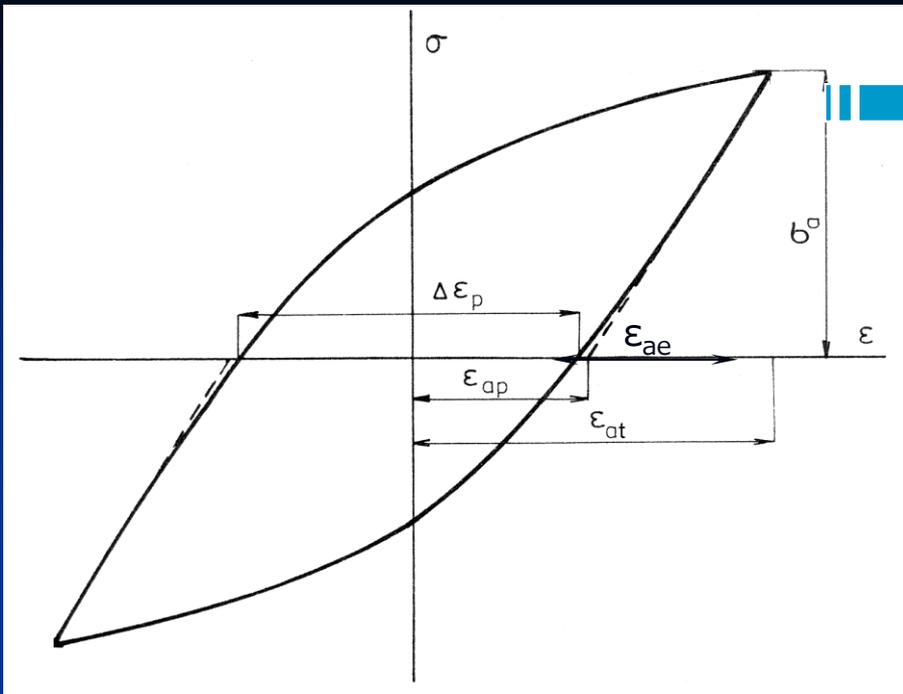
- X-Y plotter ←
- digital reading - PC – more accurate in comparison with



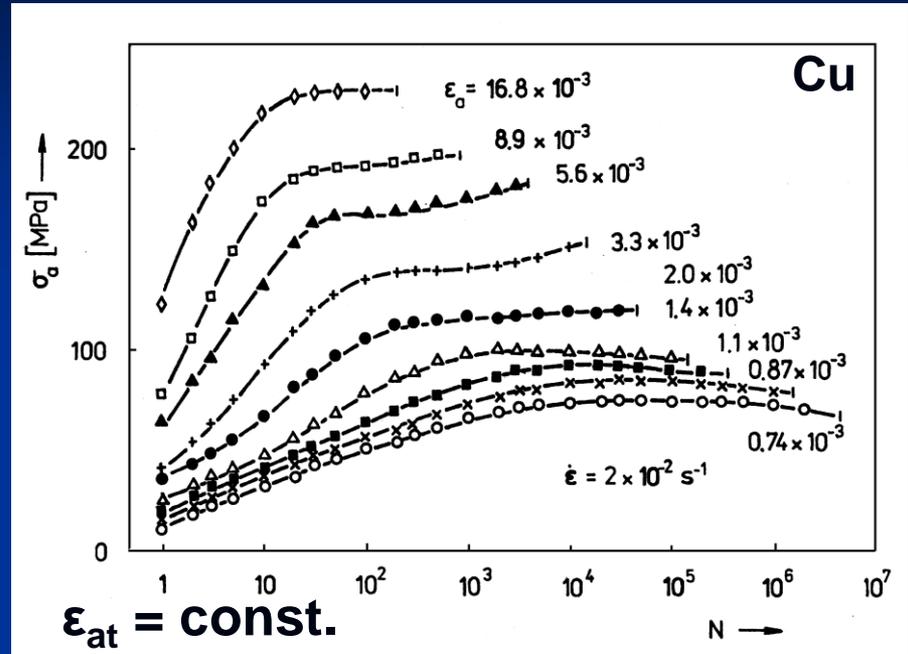
Reading of unidirectional strain (elongation):

- inductive transducer (inductor and movable core) – direct method (digital reading - PC)

- Displacement of hysteresis loops (hysteresis loop is sliding along  $\text{axe}_1, X$ )

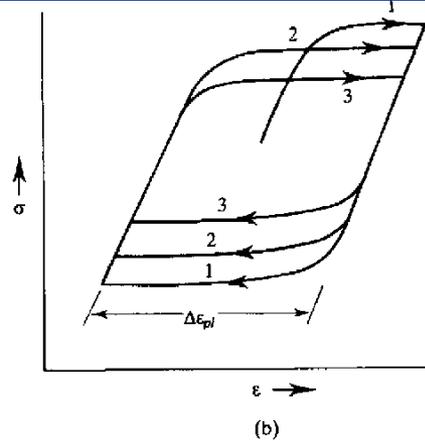
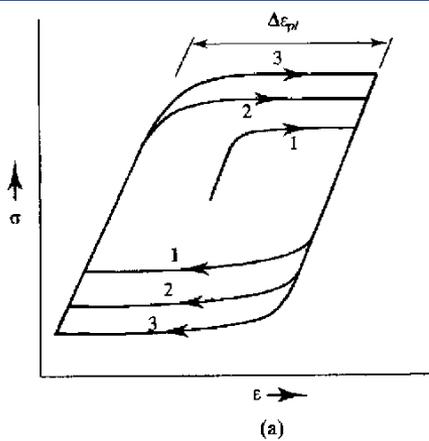


Softening/hardening curves  
Cyclic stress-strain curve

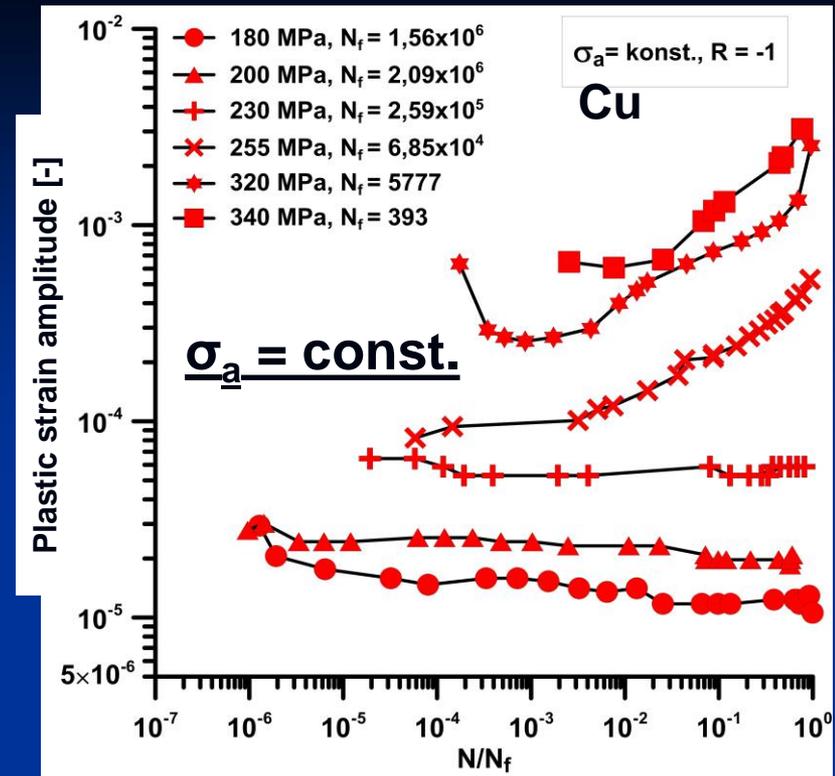
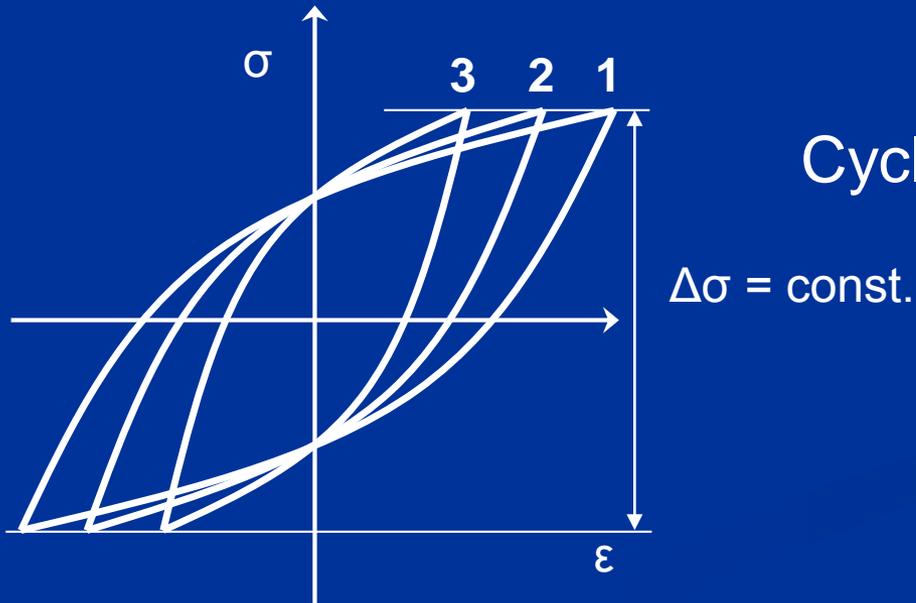
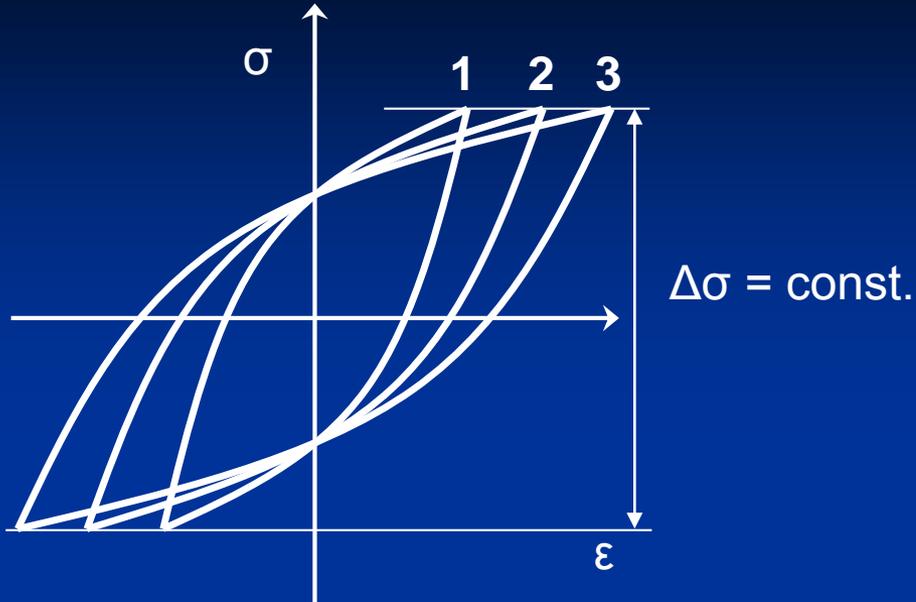


Manifestation of cyclic hardening

Manifestation of cyclic softening



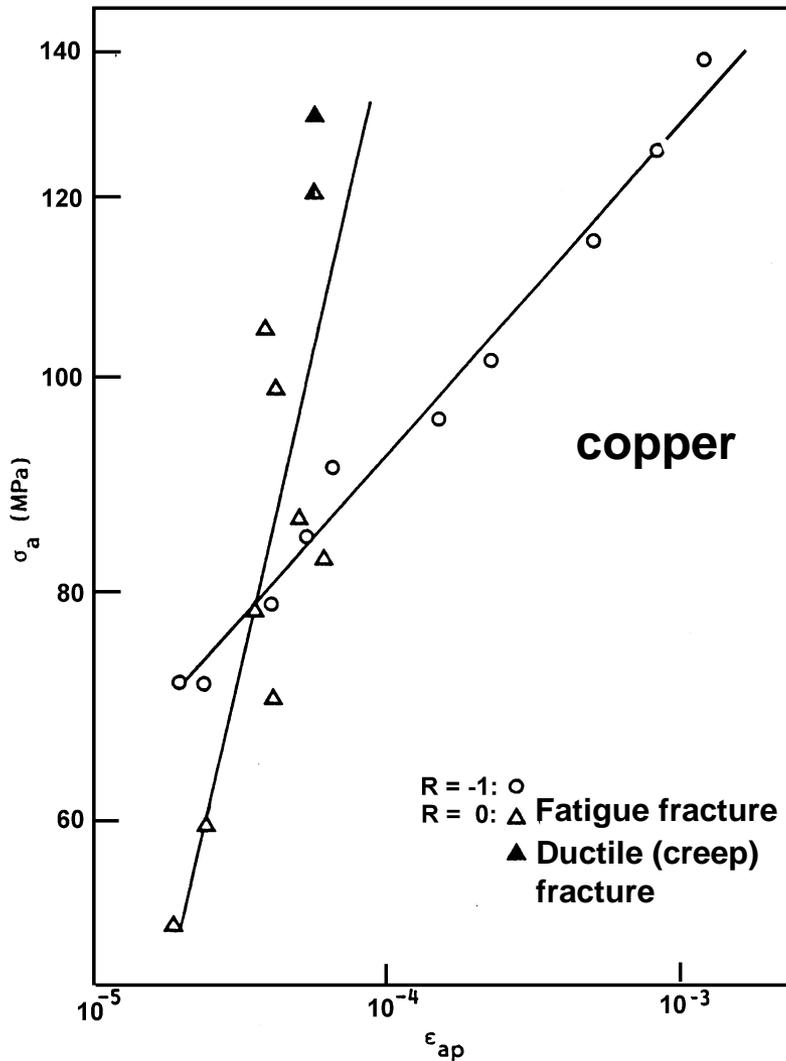
# Cyclic softening



# Cyclic hardening

# Cyclic stress-strain curve (CSSC)

Description of the cyclic plastic response of material



Available for both type of loading (load/strain control):

- from the saturated states  
( $\epsilon_{ap,sat} / \sigma_{a,sat}$ )

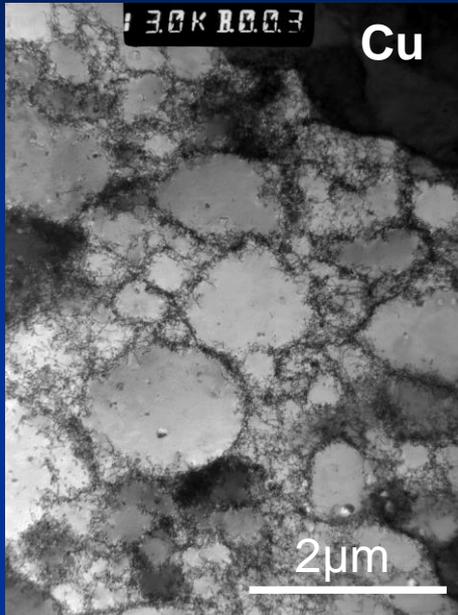
- for  $\frac{1}{2} N_f$  in the case of no saturation  
( $\epsilon_{ap,50\%} / \sigma_{a,50\%}$ )

power law form is often used for description of CSSC:

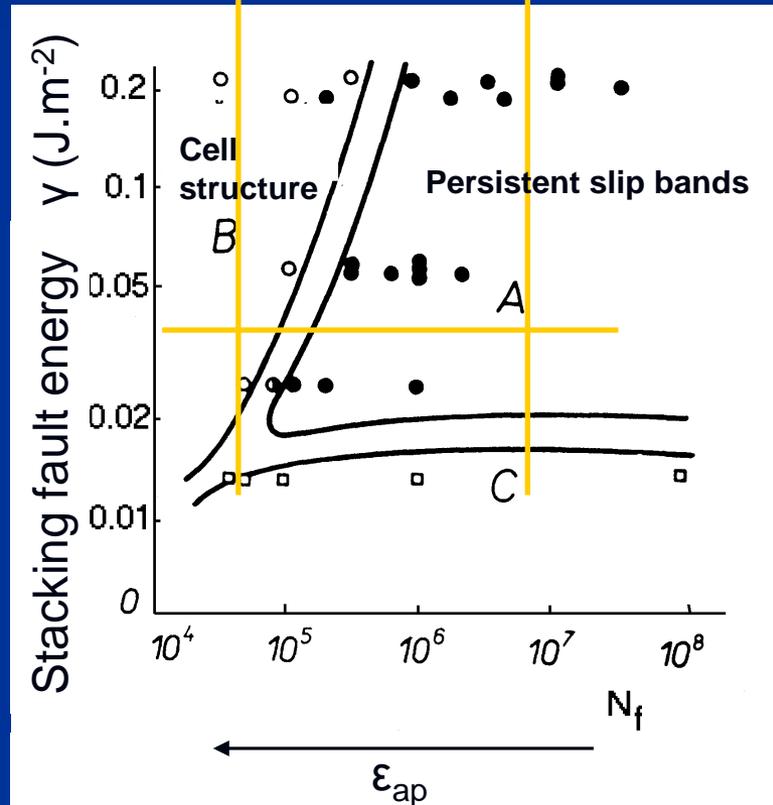
$$\sigma_a = K' (\epsilon_{ap})^{n'}$$

$K'$  - cyclic hardening coefficient  
 $n'$  - cyclic hardening exponent

# Development of specific dislocation (sub)structure during loading



FCC metals



$\gamma$  [J.m<sup>-2</sup>]

Au	0,03
Cu	0,04
Al	0,14
Ni	0,2

# Initiation of fatigue cracks

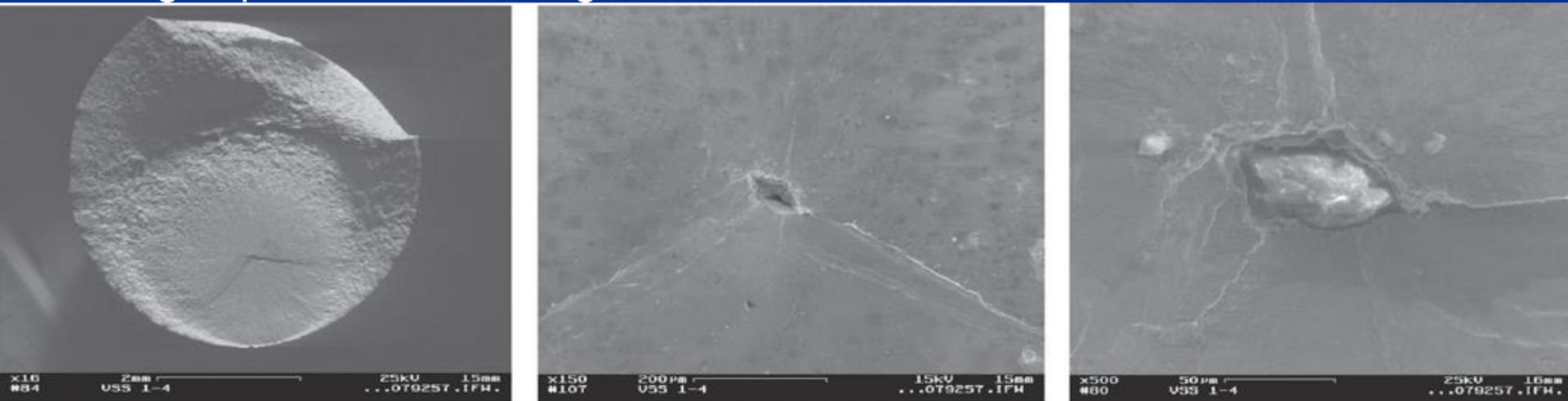
## Homogeneous materials:

- surface initiation (specimen, component)

## Inhomogeneous materials:

- interface between hardened surface layer and “softer” matrix (e.g. nitrocarburized steel etc.)
- Interface between inclusion and matrix (influence of inclusions – purity of pure metals and alloys)

For materials with prior cracks – it is assumed that initiation phase of fatigue process is missing



Basic type of crack initiation is focused to fatigue bands

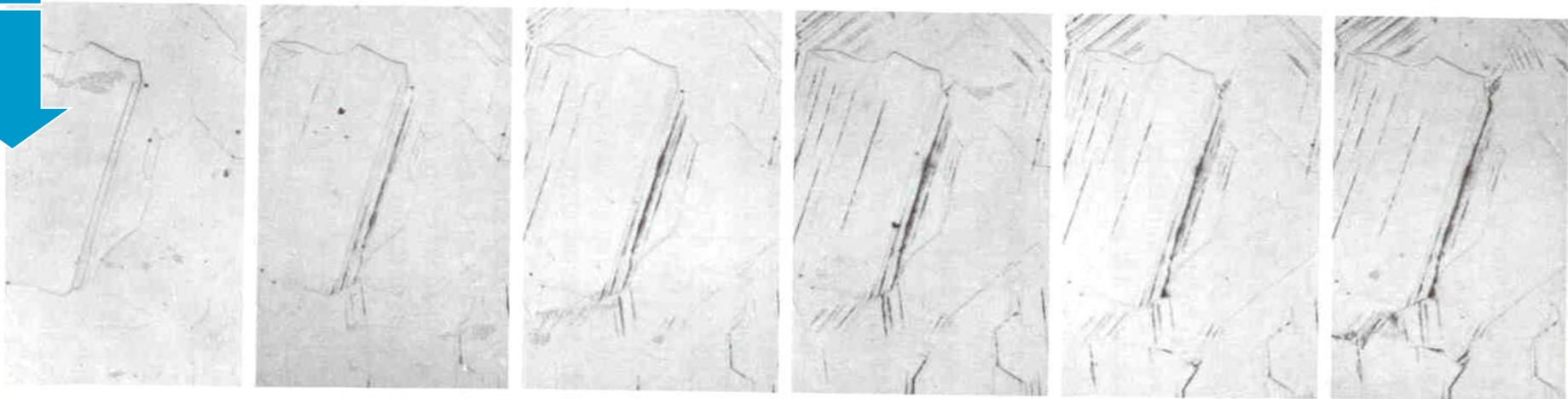
# Crack initiation in persistent slip bands

Fatigue bands – manifestation of cyclic plastic deformation

- evolution will start at the end of the stage of mechanical properties changes

- progressive evolution of fatigue bands with increasing  $N_f$

- surface relief evolution – intrusion/extrusion, intensity of bands is dependent on parameters of loading ( $\sigma_m = 0$ ,  $\sigma_m > 0$ )



$N = 0$

$10^4$

$2 \times 10^4$

$6 \times 10^4$

$10^5$

$2 \times 10^5$

$\sigma_a = 137 \text{ MPa}$ ,  $N_f \cong 1.1 \times 10^6$

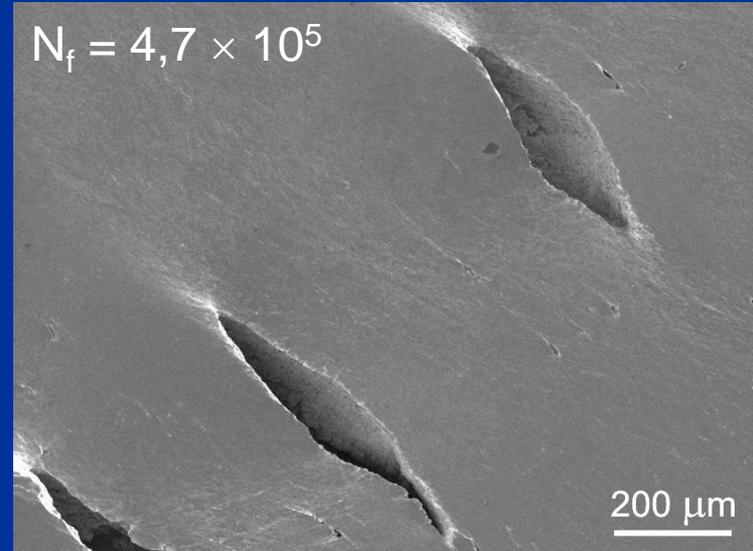
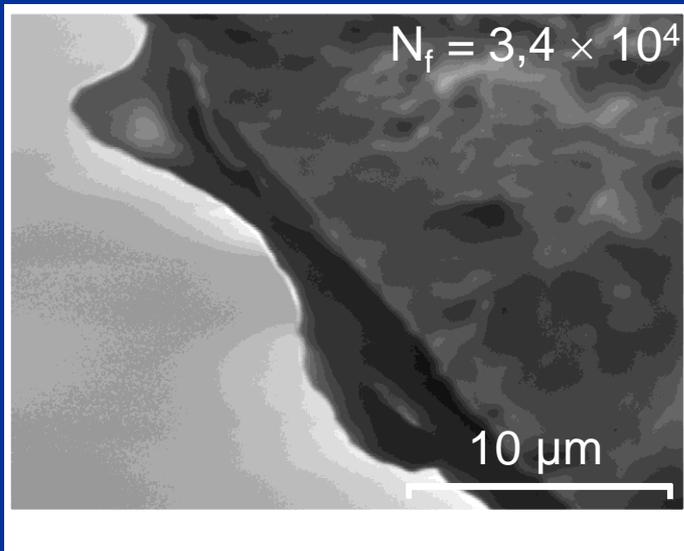
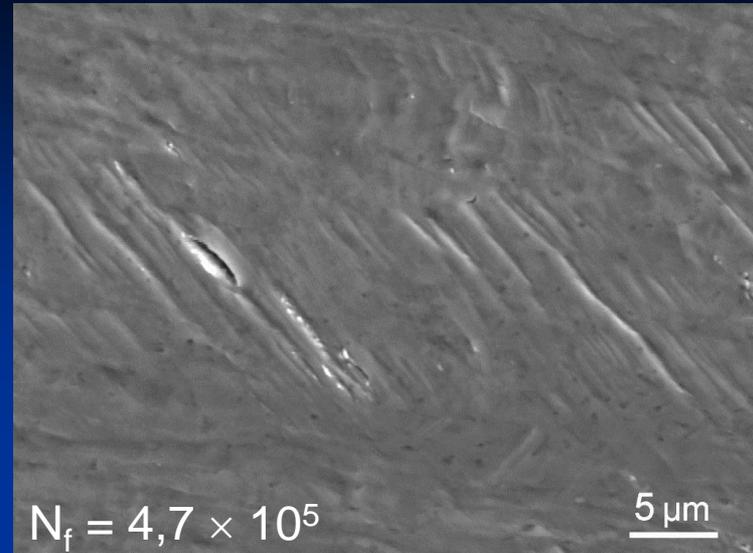
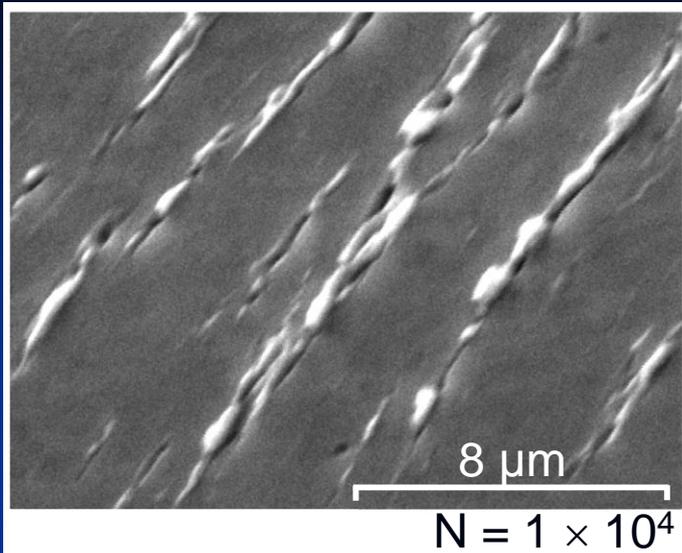
**Loading direction**



50  $\mu\text{m}$

**70Cu-30Zn**

# Persistent slip bands



$R = -1$   
 $\sigma_a = 255 \text{ MPa}$

$R = 0,1$   
 $\sigma_a = 160 \text{ MPa}$

## Stage of crack initiation:

$N_0$  – number of cycle required for crack initiation

$N_f$  – number of cycles to failure

Relative number of cycles  $N_0/N_f$

$N_0/N_f$  is dependent on:

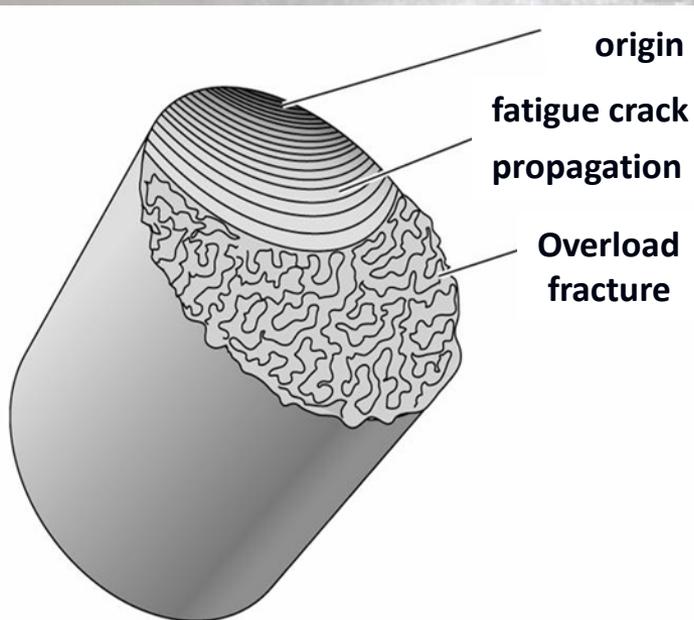
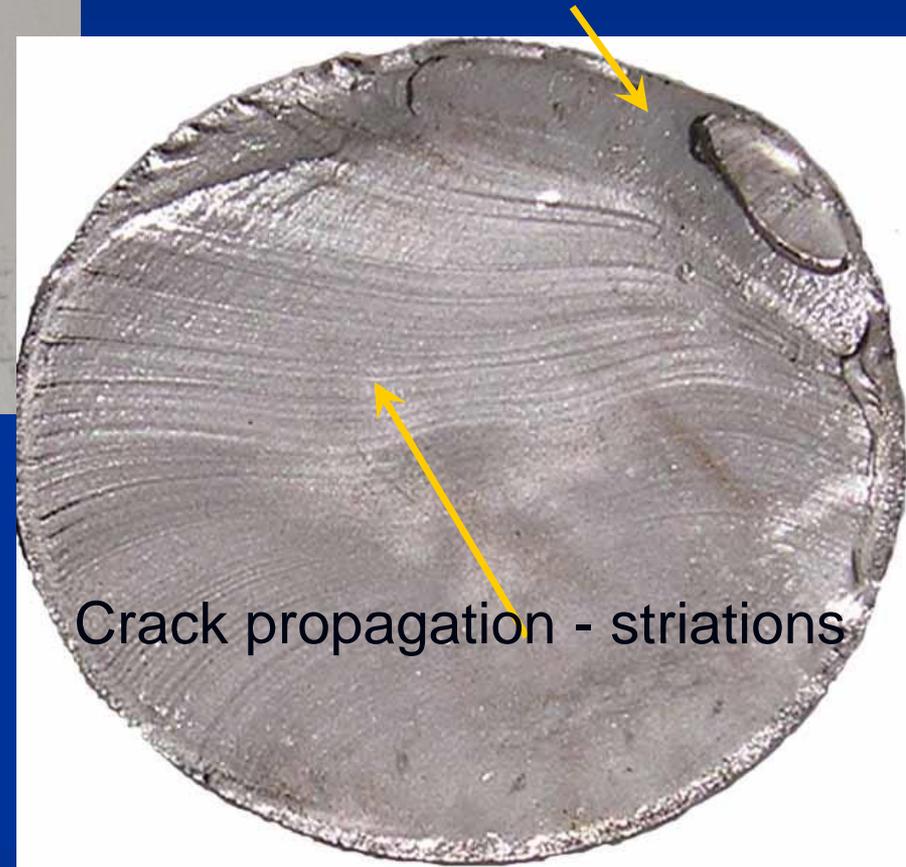
- a) Loading amplitude and asymmetry of cycle ( $N_0/N_f$  decreasing with increasing amplitude of loading)
- b) Geometry of specimen (structural parts) – influence of notches (stres concentrators)
- c) Microstructure (inclusions, hard locations)
- d) Quality / roughness of surface layer (roughness, residual stresses, contact pitting, surface hardening)
- e) Environment (corrosion pittings)

# Fatigue fracture

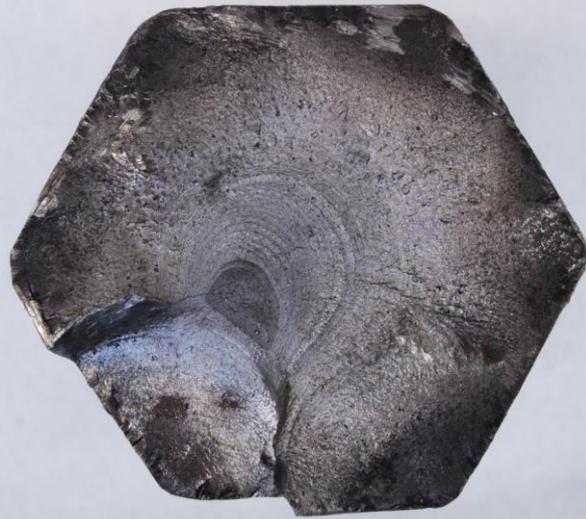
Piston of chipping hammer



overload



Shaft of tire crasher



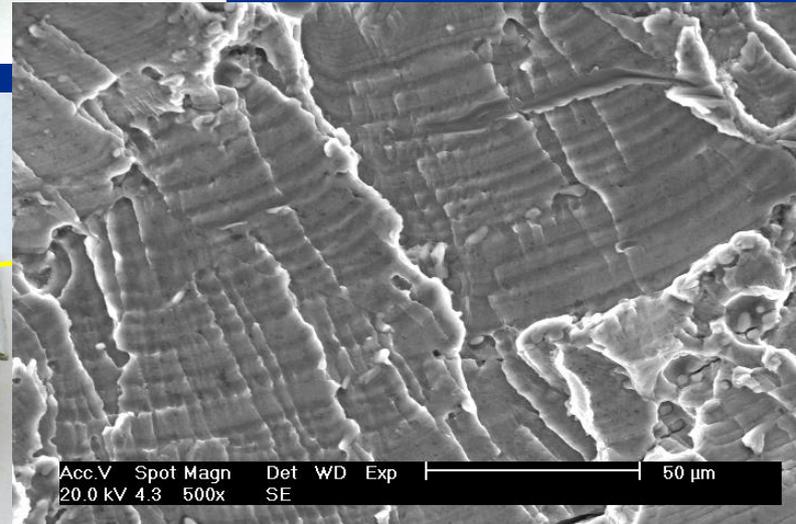
Shaft of pressing machine

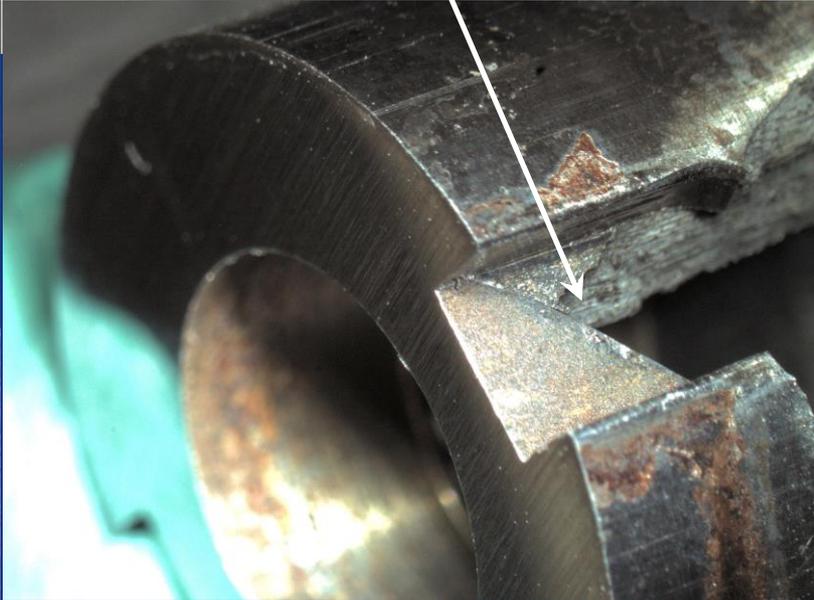
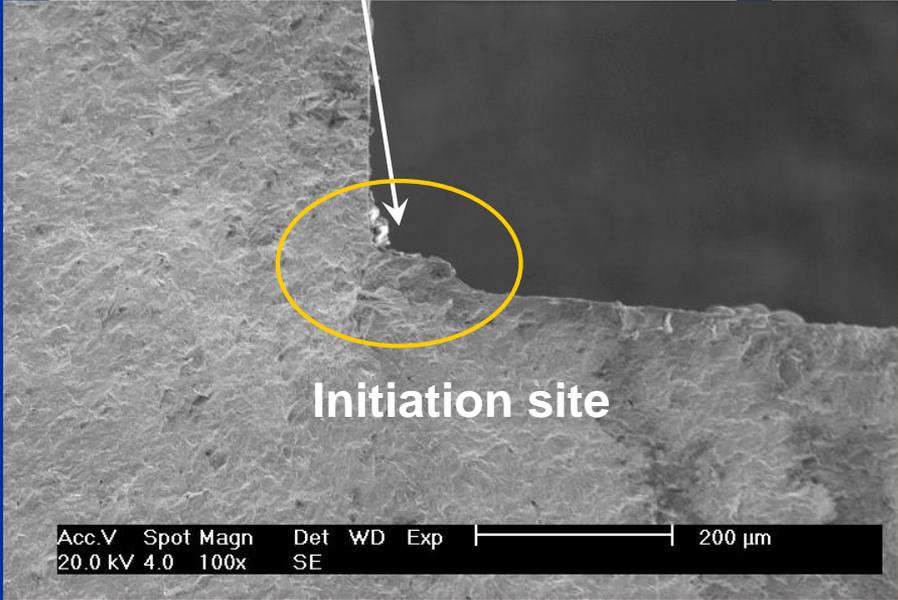


Crankshaft

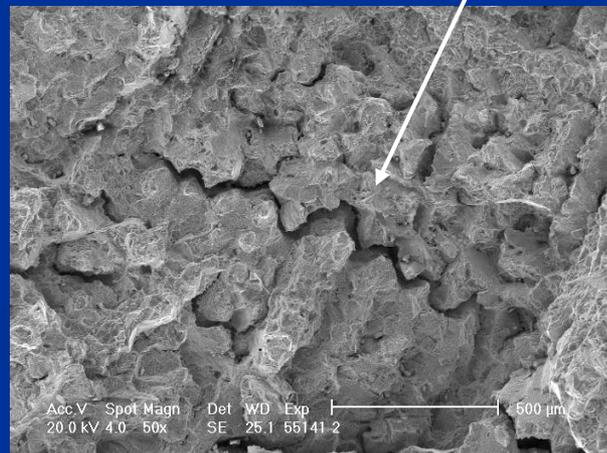
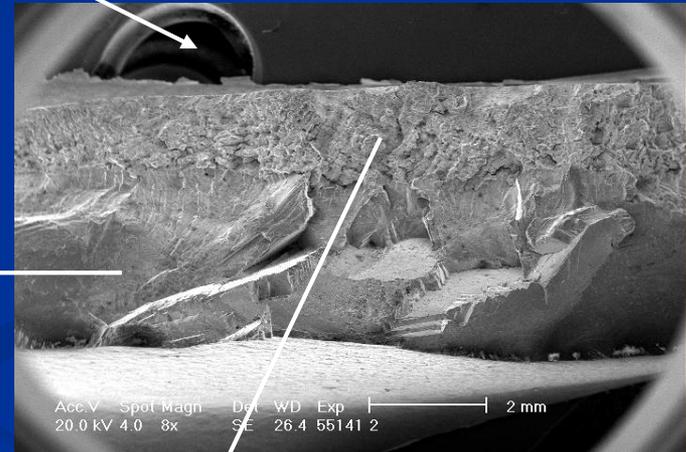
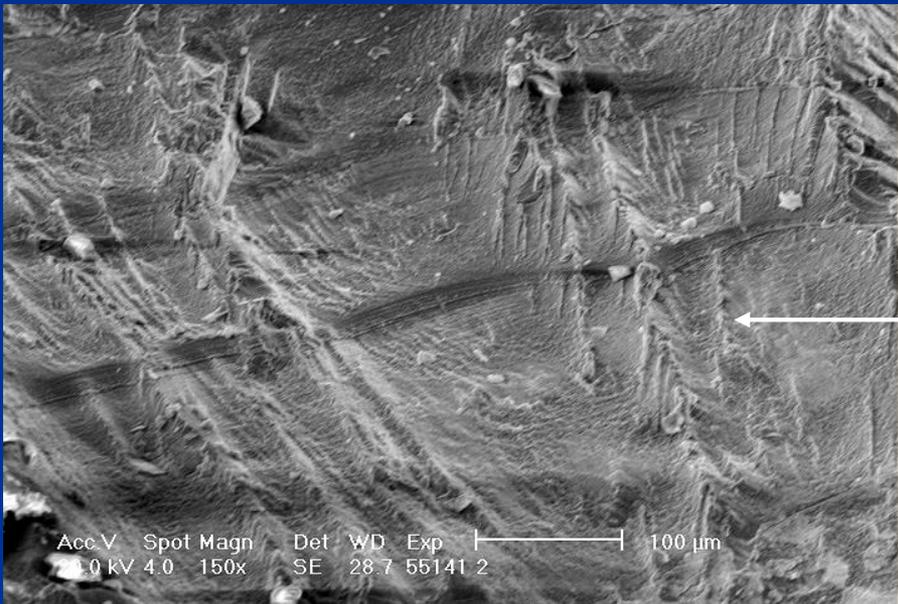
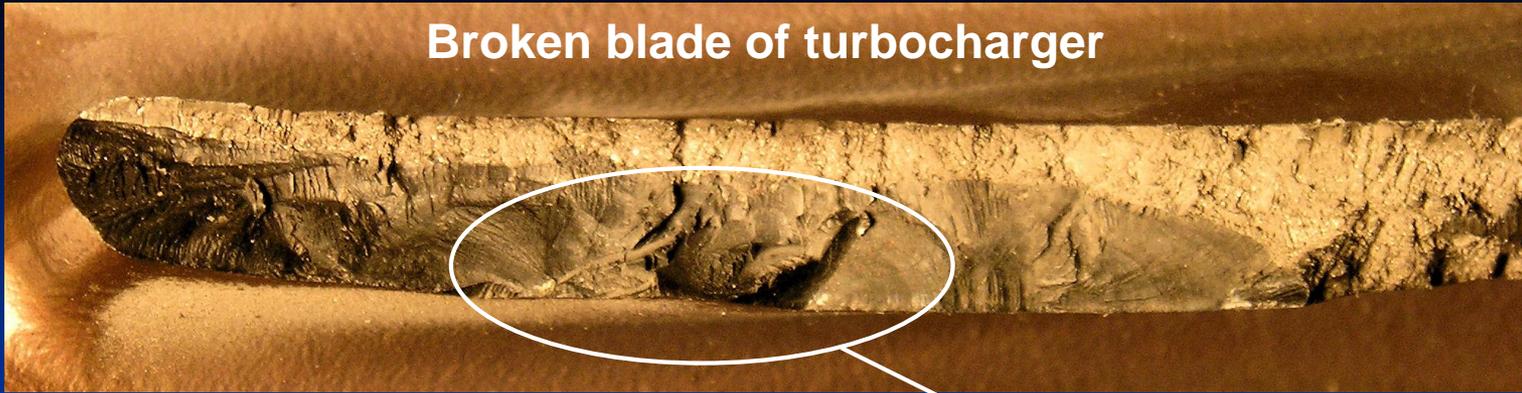


Compressor wheel blade

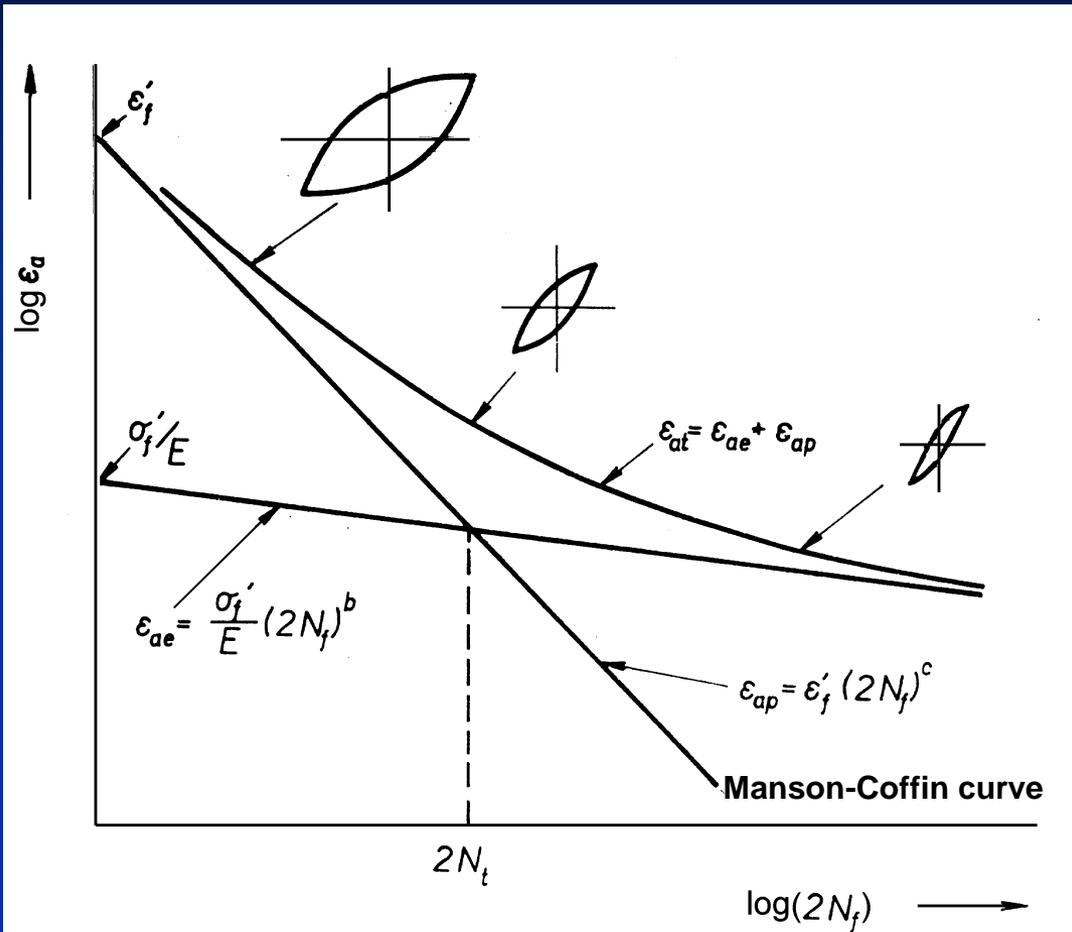




# Broken blade of turbocharger



# Manson – Coffin curve



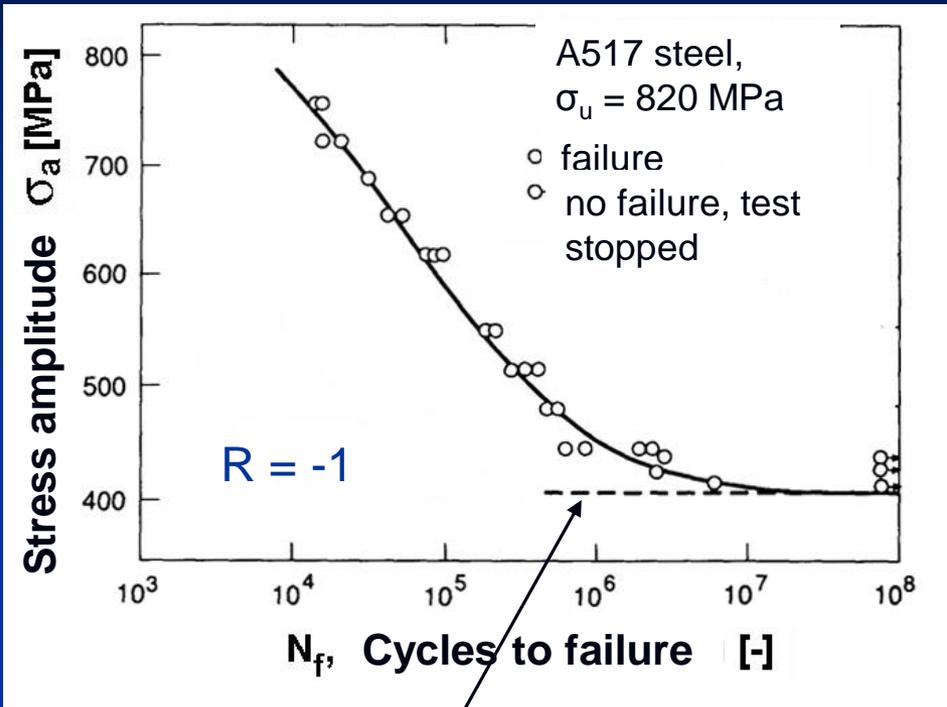
fatigue ductility exponent

$$\epsilon_{ap} = \epsilon'_f (2N_f)^c$$

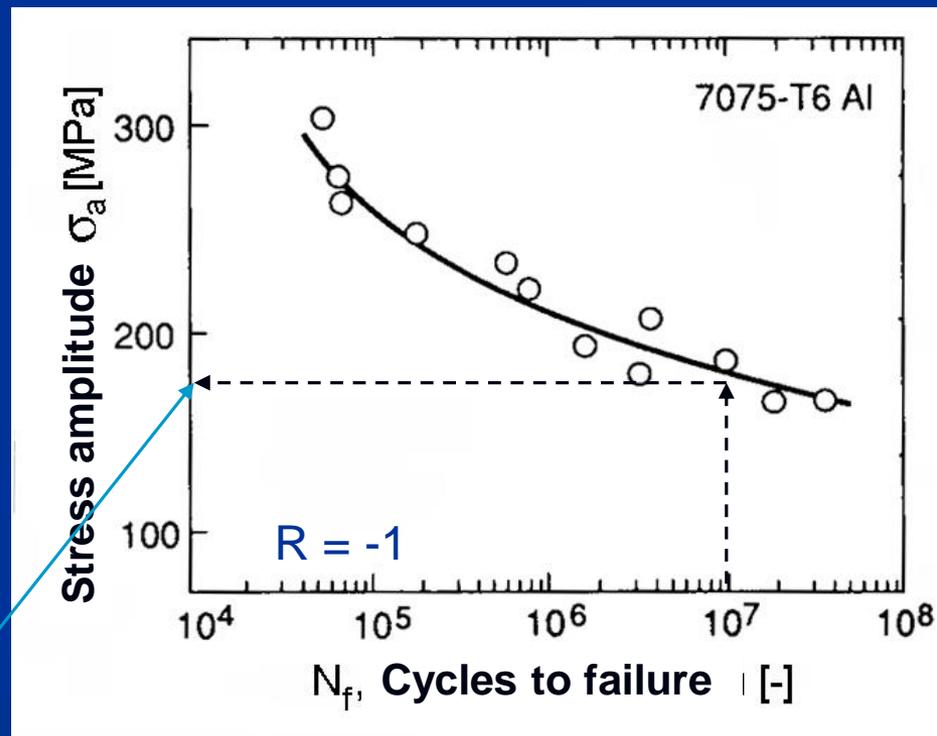
fatigue ductility coefficient

$$\epsilon_{at} = \epsilon_{ae} + \epsilon_{ap} = \frac{\sigma'_f}{E} (2N_f)^b + \epsilon'_f (2N_f)^c$$

# (S-N curve)



Permanent fatigue limit



Fatigue limit for given  $N_f$

Wöhler:

$$\sigma_a = AN_f^B$$

fatigue strength exponent

Basquin:

$$\sigma_a = \sigma'_f (2N_f)^b$$

fatigue strength coefficient

$$A = 2^b \sigma'_f, B = b$$

Fatigue life curves obtained for different loading regime ( $\sigma_a = \text{const.}$ ,  
 $\varepsilon_{ap} = \text{const.}$ )

It is possible to convert between one another through cyclic stress strain curve (CSSC).

$$\sigma_a = K' (\varepsilon_{ap})^{n'}$$

$$\sigma_a = \sigma'_f (2N_f)^b$$

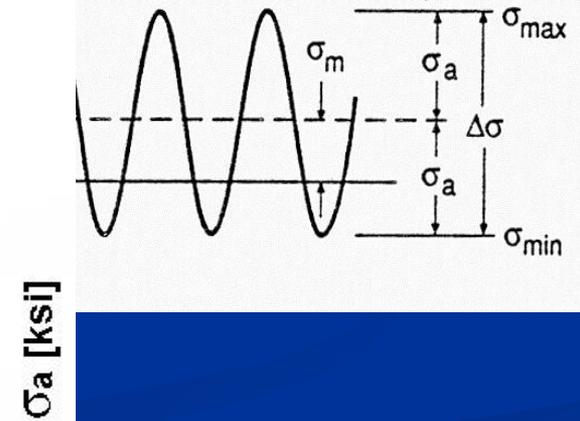
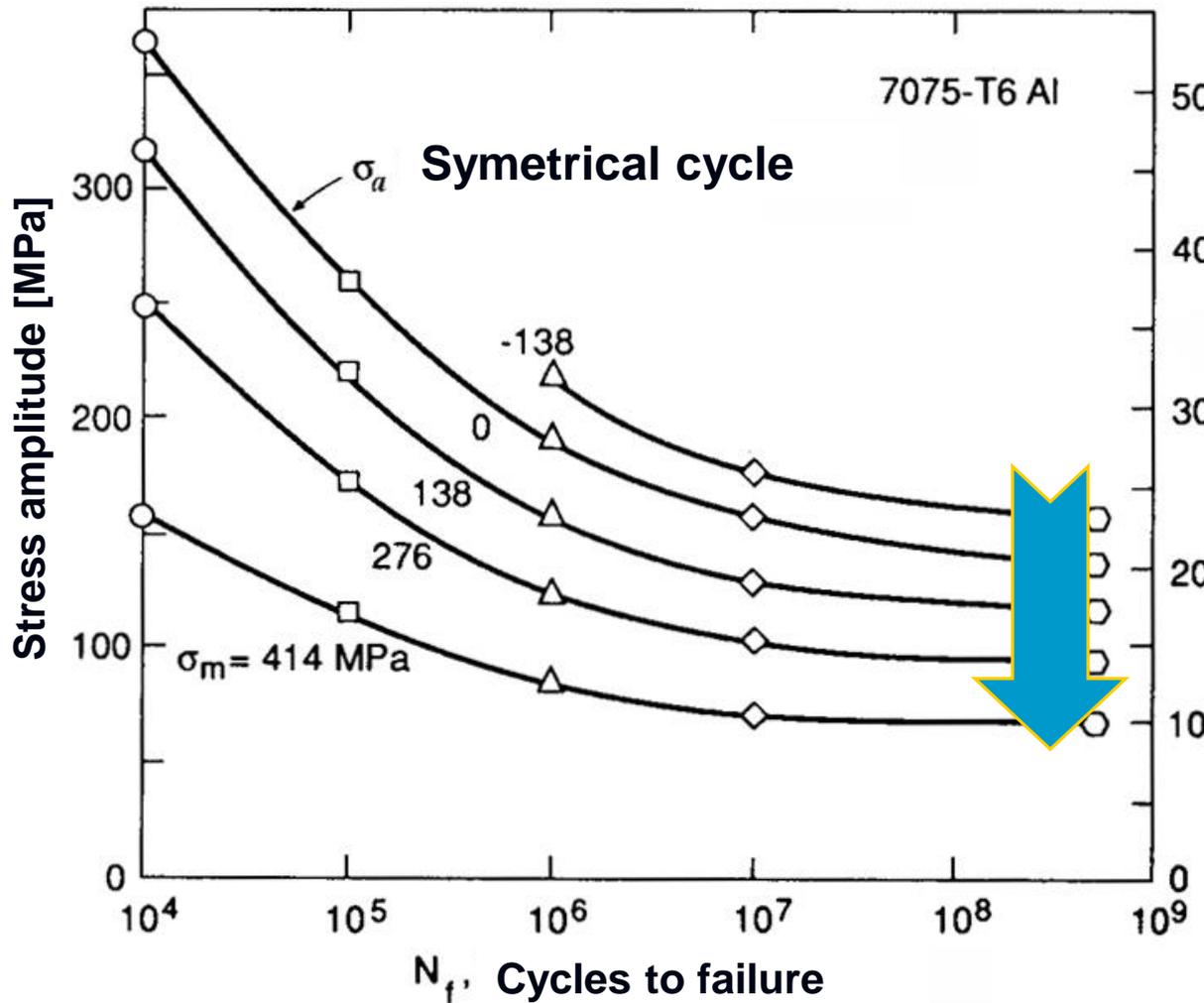
$$\varepsilon_{ap} = \varepsilon'_f (2N_f)^c$$

Parameters of fatigue life curves are not independent:

$$K' = \frac{\sigma'_f}{\varepsilon_f'^{n'}}$$

$$b = n'c$$

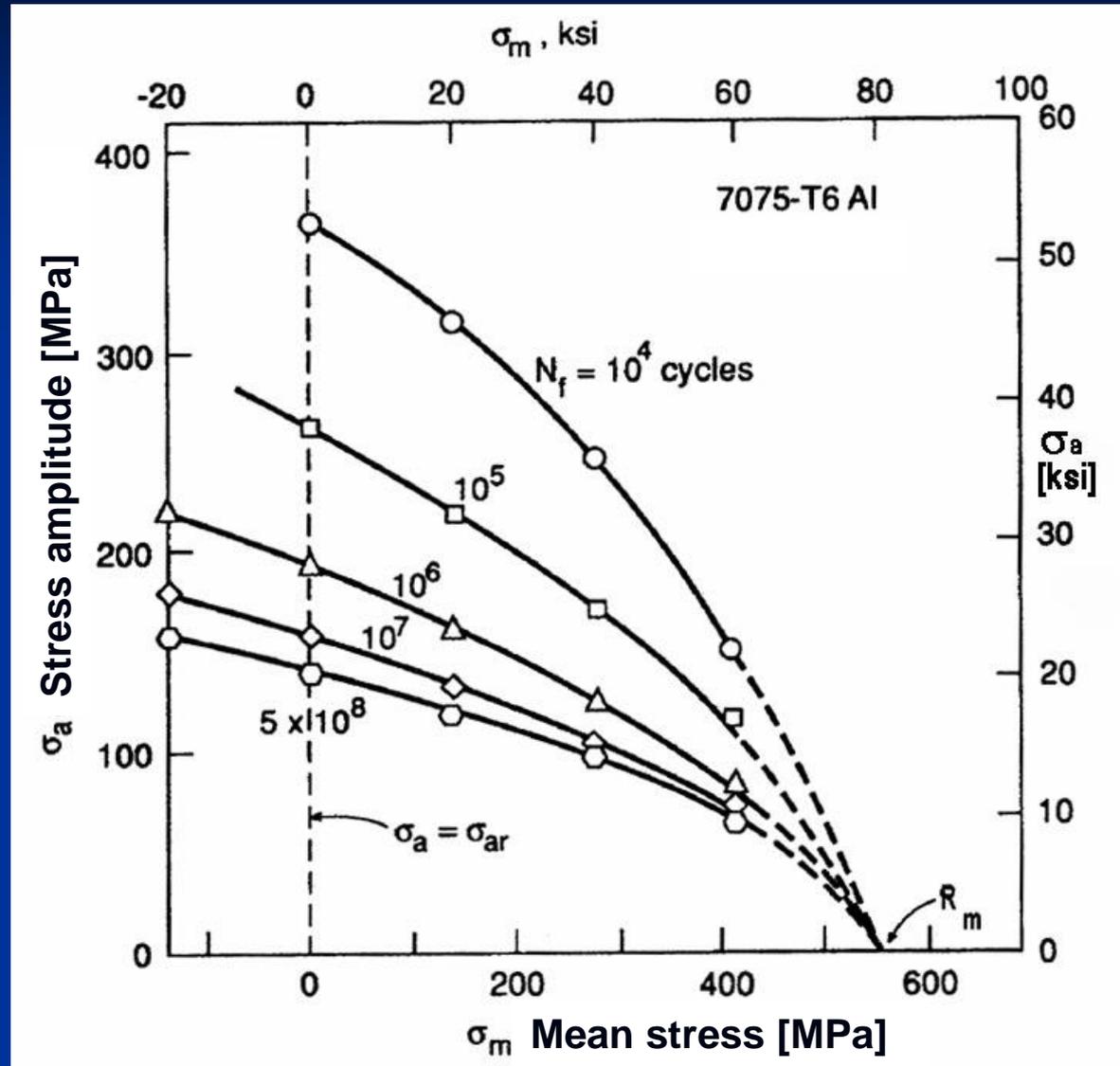
# 5) Influence of mean stress



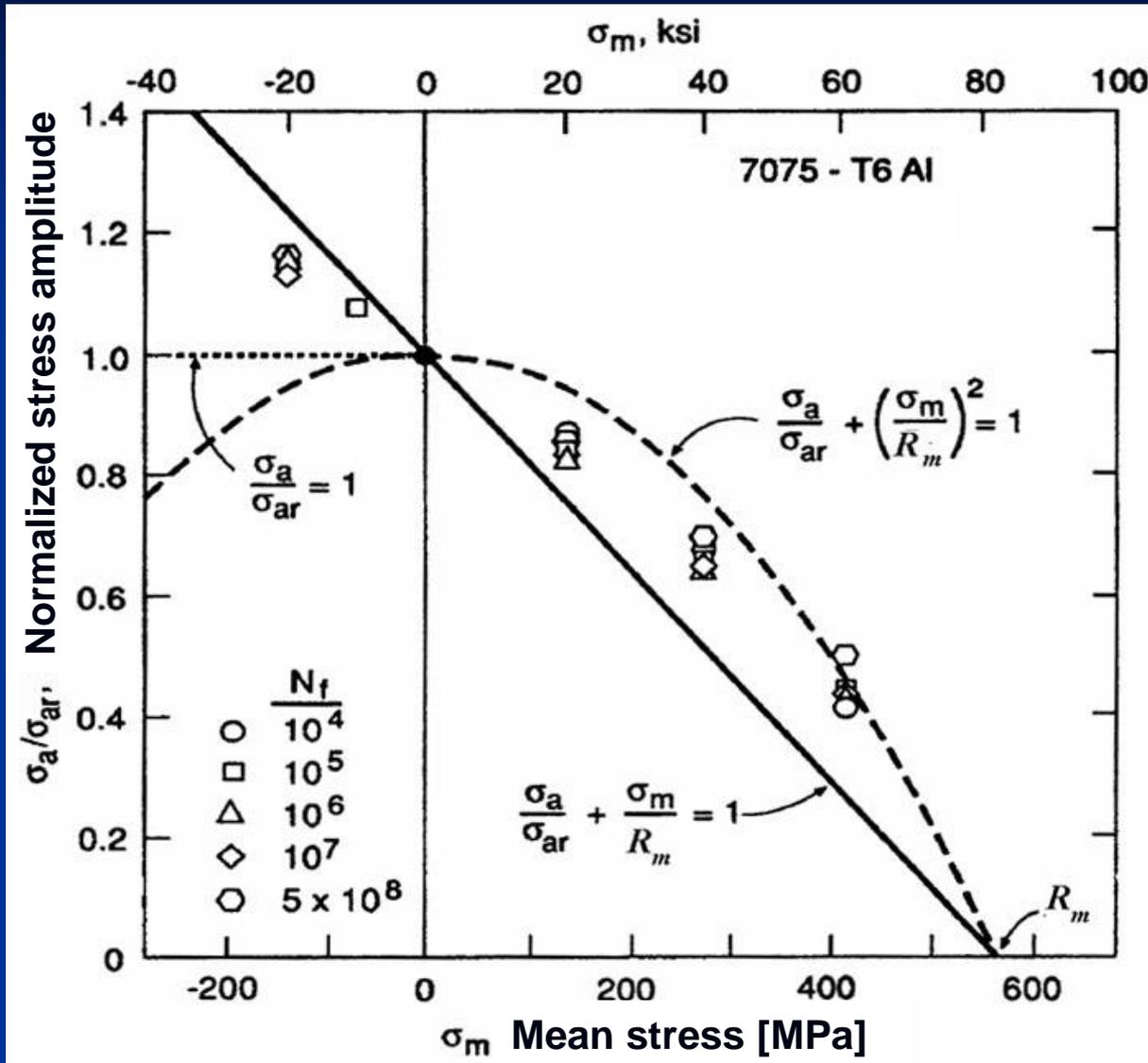
# Constant-life diagram

$$R_m = \sigma_u$$

Ultimate  
tensile  
strength



# Normalized Amplitude-Mean Stress Diagram



## Goodman

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$

$\sigma_a$  – stress amplitude of common cycle

$\sigma_{ar}$  – stress amplitude of symmetrical (regular) cycle

$\sigma_u$  – ultimate tensile strength

$\sigma_e$  – yield strength

## Gerber

$$\frac{\sigma_a}{\sigma_{ar}} + \left( \frac{\sigma_m}{\sigma_u} \right)^2 = 1$$

## Soderberg

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_e} = 1$$

Wöhler:

$$\sigma_{ar} = AN_f^B$$

Basquin:

$$\sigma_{ar} = \sigma'_f (2N_f)^b$$

Goodman:

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$



$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}}$$

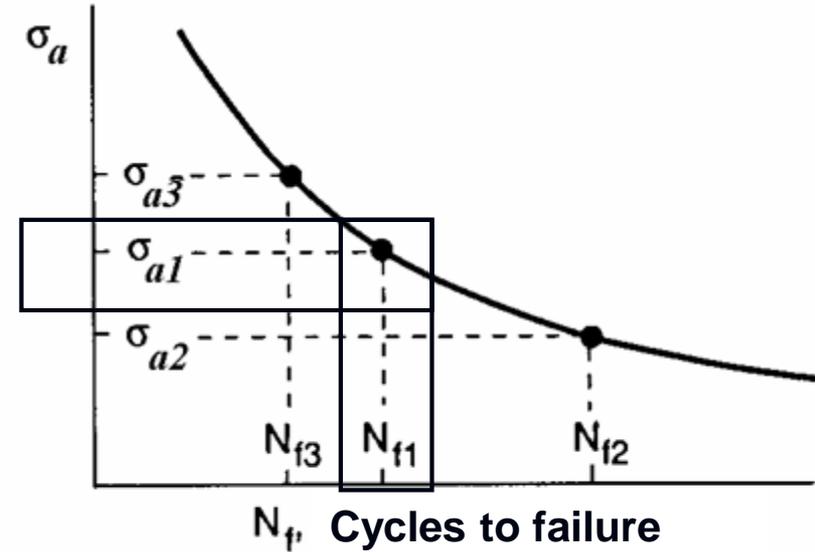
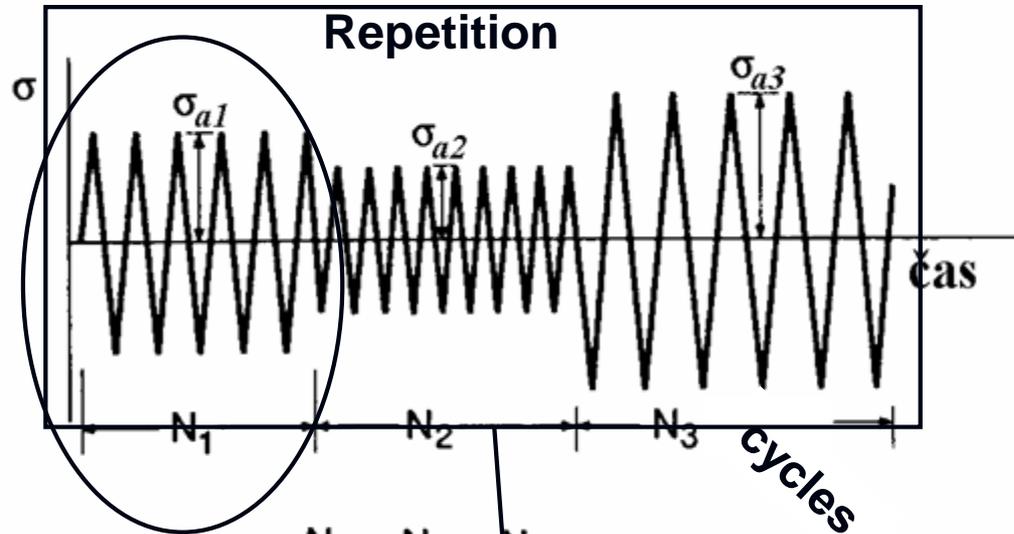
$$\sigma_a = A \left( 1 - \frac{\sigma_m}{\sigma_u} \right) (N_f)^B$$

$$\sigma_a = \sigma'_f \left( 1 - \frac{\sigma_m}{\sigma_u} \right) (2N_f)^b$$

For more accurate description  $\sigma'_f$  is used in place of  $\sigma_u$

$$\sigma_a = (\sigma'_f - \sigma_m) (2N_f)^b$$

# The Palmgren-Miner Rule



$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \dots = 1$$

Failure is expected when

$$\frac{N_1}{N_f^1} + \frac{N_2}{N_f^2} + \dots = \sum_{j=1}^k \frac{N_j}{N_f^j} = 1$$

$$B = \frac{1}{\frac{N_1}{N_f^1} + \frac{N_2}{N_f^2} + \frac{N_3}{N_f^3}}$$

Fraction of the life

## Literature:

- [1] DOWLING, N.E. *Mechanical behavior of materials*. Engelwood Cliffs, USA: PRENTICE-HALL, 1993, 773 s., ISBN 0-13-026956-5.
- [2] SURESH, S. *Fatigue of Materials*. 2<sup>nd</sup> edition. Cambridge, UK: Cambridge University Press, 2003. 679 s. ISBN 0-521-57847-7.
- [3] SCHÜTZ, W. A history of fatigue. *Engineering Fracture Mechanics*, 1996, Vol. 54, No. 2, s. 263–300. Available from www: <http://www.sciencedirect.com/>.